

# Online Adaptive Frequency Hopping

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## Abstract

Adaptive frequency hopping is one way to maximize the utilization of the wireless spectrum. Yet, when the environment itself is changing, the frequency at which the radio senses can become increasingly less optimal. By having the radio create a model of the environment based off of the sensing data, it is possible to achieve high data rates when the spectrum is not being heavily utilized and maintain a low level of interference at times when it is. The radio was modeled both mathematically and run in simulations. The outcomes of these tests were compared with existing standards such as Bluetooth (random frequency hopping) and IEEE 802.22 (fixed sensing rate). In order to evaluate data rate and interference simultaneously, a metric was created that combined them by taking the product of data rate and  $(1 - \text{interference})$ . Overall, the online adaptive frequency hopper had a 35% increase in the combined metric over the random frequency hopper and 25% increase over the fixed sensing rate radio.

## I. INTRODUCTION

With the increased popularity of smartphones and other wireless devices, global demand for wireless spectrum has increased dramatically since its first use in radio and television. Yet, while demand for such resources is on the rise, organizations such as commercial carriers and government agencies remain limited in the amount of spectrum allocated for their use. As a result, increases in data rate due to reallocation of spectrum are relatively infrequent. However, it is possible to create significant increases in data rate by changing how specific bands of frequencies or channels are managed.

While certain channels may be continually allocated to services such as television stations or commercial carriers, these channels are not always in use by the services they were allocated to. For instance, a local station could have finished broadcasting its program for the day, or a conference call could have recently ended. This fragmentation in the use of wireless spectrum provides gaps for a user who is not licensed to use a specific channel, or secondary user, to use the downtime of channels already licensed to a specific organization or primary user. This is known as opportunistic access and is only allowed if the secondary user provides a minimal amount of interference to the primary user.

Some wireless policies such as Bluetooth attempt to harness some of the potential of opportunistic access by continuing to change the channel on which they operate, thereby minimizing the interference to an individual channel. However, this system can be improved upon by

incorporating a specified period in which to sense the state of the system [1]. This data can then be used to determine which channels have the minimum probability of being occupied by primary users and therefore available for use by secondary users.

In a further optimization of existing protocols, a radio can use the data accumulated by previous sensing results to determine the volatility of a channel, a set of values representing how often the channel changes. Allowing knowledge of the environment to influence the frequency at which a radio senses allows the radio to excel in the two categories of environments: when the channel availability is changing rapidly and when channel availability is changing slowly. When channel availability is changing rapidly, frequent sensing allows the radio to ensure that it is not still using channels past the period of availability. On the other hand, when channel availability is changing slowly, decreased sensing frequency allows the radio to harness the full potential of longer periods of availability by not wasting time sensing [2].

## II. RADIOS

In the experiment, three types of radios were tested. These included: a random frequency hopper (which would randomly choose its frequency without sensing), a fixed adaptive frequency hopper (which would choose its channels from the available channels in the previous sensing rounds but have a constant sensing frequency), and an online adaptive frequency hopper (which would choose its channels from the available channels in the previous sensing rounds and use its sensing data to determine the optimal sensing frequency). Both the random frequency hopper and the fixed adaptive frequency hopper acted as the control in this case, representing existing technologies such as Bluetooth and IEEE 802.22 [3], respectively. The fixed adaptive frequency hopper had a period of 30 seconds to collect data between sensing periods like IEEE 802.22. Additionally, the experiment examined the effectiveness of two methods of determining which channels to use for adaptive frequency hoppers. The first method, "best", simply used the channels that were available in the last frame as the channels for the next frame. The second method, "double-back", used only channels that were available in both the previous two frames as the channels for the next frame.

Three metrics were used to judge the effectiveness of each protocol. These included data rate, interference, and a combination of the two. Specifically, data rate and interference were measured as the percentage of time that the radio was sending data or interfering, respectively.

This was done to accommodate differences in physical hardware that would use this protocol. If a device had a theoretical maximum of 50 Mbps and a percent data rate of .5, the actual data rate of the device would be 25 Mbps, the result of taking the product of the two numbers. Given  $N$  channels, the time spent interfering with a specific primary user is given by  $\frac{interference*time}{N}$ , where *interference* is the percent interference and *time* is the total time spent communicating data. The final metric, which attempted to combine both data rate and interference, was calculated by taking the product of data rate and (1-interference). In order to evaluate these metrics for the various protocols, both mathematical models and python simulations were developed. In addition to random conditions, simulations were run that modeled spectrum fluctuations over the cycle of a day and in times of an emergency [4].

### III. MATHEMATICAL MODEL

Since at any point in time, a channel can either be in use by a primary user or available for use by a secondary user, a channel can be represented by a time-dependent function  $f(t)$ , where a value of 1 represents availability for use and a value of 0 represents the channel being occupied. Additionally, since the channel is in use for particular lengths of time (e.g. a cell phone call), a channel can modeled by a series of alternating periods of unavailability and availability. It is then possible to define the constants  $g$  and  $b$  which represent the likelihood of a channel changing from unavailable to available and available to unavailable, respectively. Specifically, given an exponentially decreasing probability density function  $f(t) = \rho e^{-\rho t}$  of the duration of one of these periods being of length  $t$ ,  $g$  and  $b$  represent  $\rho$  in the corresponding equations  $f_1(t)$  and  $f_2(t)$ .

Using equations  $f_1(t)$  and  $f_2(t)$ , it is then possible to derive an equation for the probability that a channel is available at time  $t$ , given a measurement of the state being available at time 0. This function  $\pi(t)$  can be solved for through the use of renewal series. Specifically, one can write the relationship between  $\pi(t)$ ,  $f_1(t)$ , and  $f_2(t)$  as

$$\pi^*(t) = \frac{1 - f_1^*(t)}{s(1 - f_1^*(t)f_2^*(t))}$$

where the "\*" operator represents a Laplace transform from the time domain  $t$  to the frequency domain  $s$  [5].

The corresponding solution to equation 1 yields

$$\pi(t) = \frac{b + ge^{-(b+g)t}}{b + g} \quad (1)$$

Likewise, there exists a similar function  $\phi(t)$ , which represents the probability that a channel is available at time  $t$  given that the channel was unavailable at time 0. The solution to this is

$$\phi(t) = \frac{b(1 - e^{-(b+g)t})}{b + g} \quad (2)$$

These functions will be used later in calculating both optimal sensing rate as well as expected data rate and interference. Additionally, note that by taking the limit of either of these two functions yields the probability that a channel is available at any time.

$$p = \lim_{t \rightarrow \infty} \phi(t) = \frac{b}{b + g} \quad (3)$$

For the sake of running a simulation, a series of distinct periods of time for unavailable and available had to be calculated. In order to transform the distribution of the random number generator found on the test computer (on which any number between 0 and 1 has an equal chance of being selected) to the distributions required by  $f_1$  and  $f_2$ , inverse transform sampling was used. Let the variable  $r$  represent the result of the number generator (which is on the range  $[0, 1)$ ) and the variable  $L$  represent the length of one of these periods. Therefore, with  $\rho$  representing either  $g$  or  $b$ ,

$$\begin{aligned} r &= \int_0^L \rho e^{-\rho t} dt \\ r &= 1 - e^{-\rho L} \\ 1 - r &= e^{-\rho L} \\ \ln(1 - r) &= -\rho L \\ L &= -\frac{\ln(1 - r)}{\rho} \end{aligned} \quad (4)$$

For each channel  $i$  in the simulation, a series of alternating available and unavailable sequences were calculated with the aforementioned method to create the corresponding function  $C_i(t)$ , which represents the availability of channel  $i$  at time  $t$ . The remaining pieces of the derivation of data rate and interference can be found in the appendix.

#### IV. SIMULATION

The simulation was run as follows. Each radio was given a specified time to tune to a specific channel of their choosing known as the  $t_s$  during which the radio would either collect data or cause interference, depending on the availability of the channel selected. Additionally, the radios which could sense were given a period of time known as the  $t_c$  at the end of which they would have gathered the availability of all channels. This time would neither add to their interference or data rate, but by inaction would cause both to decrease. Further, a probability of incorrectly sensing an unavailable channel to be available,  $R_p$ , and a probability of incorrectly sensing an available channel to be unavailable,  $R_n$ , were also taken into account to better simulate physical devices. The random adaptive frequency hopper would continually hop without sensing, while the other radios would have one period to sense and  $H$  hops to collect data or interfere. Together, the one sensing period and  $H$  hops make a frame.

#### V. OPTIMIZATIONS

For the purposes of this experiment, the performance of a radio was measured by the product of the data rate and 1 - interference. This calculation works well since most of the calculated functions for data rate and interference have a curve which show a fairly clear region of being optimal that the performance metric finds. Since all aspects of data rate and interference calculations were fixed except for the hops per frame, the radio would maximize the performance with respect to its hops per frame and use the result as the number of hops to take in the next frame.<sup>1</sup>

#### VI. ESTIMATING THE ENVIRONMENT

In order to determine the optimal number of hops per frame, the environmental variables  $g$  and  $b$  have to be determined. This was done through the use of a maximum likelihood estimator. Given a set of  $S$  samples of  $N$  channels, one can identify  $(S-1)*N$  transitions.<sup>2</sup> Let the transition from available to available, available to unavailable, unavailable to available and unavailable to

<sup>1</sup>As a side note, as the performance equation had no analytical solution that could be added to the simulation, a table was computed with the hops per frames across a range of  $b$  and  $g$ .

<sup>2</sup>For the purpose of the simulation, each channel was given the same environment. However, if one would like to treat each channel individually, simply repeat these calculations for each channel, instead of using all samples as a whole.

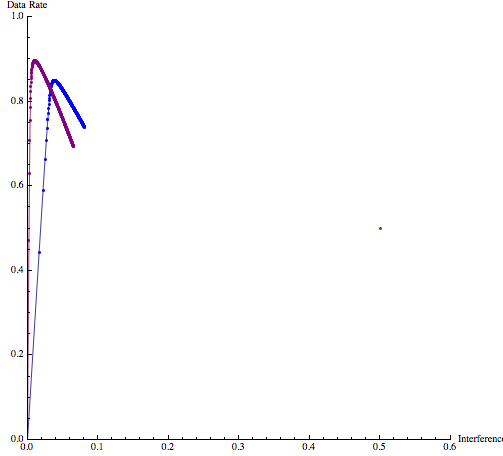


Fig. 1. Graph of data rate and interference as a function of hops per frame for double-back (purple), single-back (blue), and random (green).

unavailable be represented by  $r_{11}, r_{10}, r_{01}$ , and  $r_{00}$ , respectively. Where  $t$  represents the average time between all frames, the solution to the maximum likelihood estimator yields

$$g = \frac{(r_{00} + r_{01}) \cdot r_{11} \cdot \ln\left(\frac{(r_{00}+r_{01})(r_{10}+r_{11})}{r_{00} \cdot r_{11} - r_{01} \cdot r_{10}}\right)}{(r_{00} \cdot r_{10} + 2 \cdot r_{01} \cdot r_{10} + r_{01} \cdot r_{11}) \cdot t} \quad (5)$$

$$b = \frac{(r_{11} + r_{10}) \cdot r_{01} \cdot \ln\left(\frac{(r_{00}+r_{01})(r_{10}+r_{11})}{r_{00} \cdot r_{11} - r_{01} \cdot r_{10}}\right)}{(r_{00} \cdot r_{10} + 2 \cdot r_{01} \cdot r_{10} + r_{01} \cdot r_{11}) \cdot t} \quad (6)$$

However, this situation assumes no error in determining the values of  $r$ . In order for the equations above to be valid, one must now calculate the values of  $r$ , given some samples  $n$ , with error  $R_p$  and  $R_n$ . These equations are as follows.<sup>3</sup>

$$r_{00} = \frac{(R_n - 1)^2 \cdot n_{00} + R_n \cdot ((R_n - 1)(n_{01} + n_{10}) + R_n \cdot n_{11})}{(R_n + R_p - 1)^2} \quad (7)$$

$$r_{01} = \frac{n_{01} - R_n \cdot (n_{01} + n_{11}) + R_p \cdot ((R_n - 1) \cdot n_{00} - n_{01} + R_n \cdot (n_{01} + n_{10} + n_{11}))}{(R_n + R_p - 1)^2} \quad (8)$$

$$r_{10} = \frac{n_{10} - R_n \cdot (n_{10} + n_{11}) + R_p \cdot ((R_n - 1) \cdot n_{00} - n_{10} + R_n \cdot (n_{01} + n_{10} + n_{11}))}{(R_n + R_p - 1)^2} \quad (9)$$

$$r_{11} = \frac{(n_{11} + R_p^2 \cdot (n_{00} + n_{01} + n_{10} + n_{11}) - R_p \cdot (n_{01} + n_{10} + 2 \cdot n_{11}))}{(R_n + R_p - 1)^2} \quad (10)$$

<sup>3</sup>In the actual simulation,  $\frac{1}{(R_n + R_p - 1)^2}$  was factored out from the equations since all occurrences of values of  $r$  in the estimation of  $b$  and  $g$  are ratios.

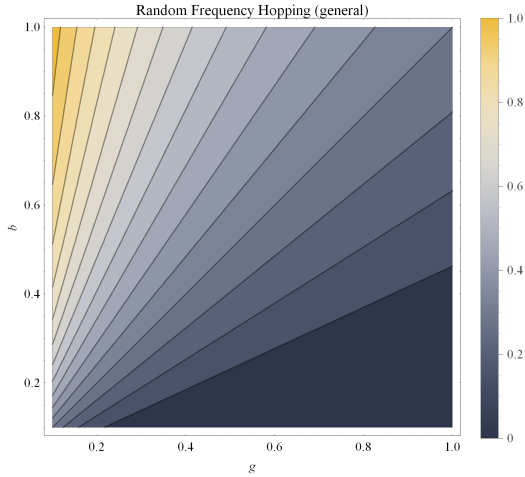


Fig. 2. Graph of the combined data rate and interference metric plotted vs time constants  $g$  and  $b$  for both the random frequency hopper. Note the large region of almost no data on the right.

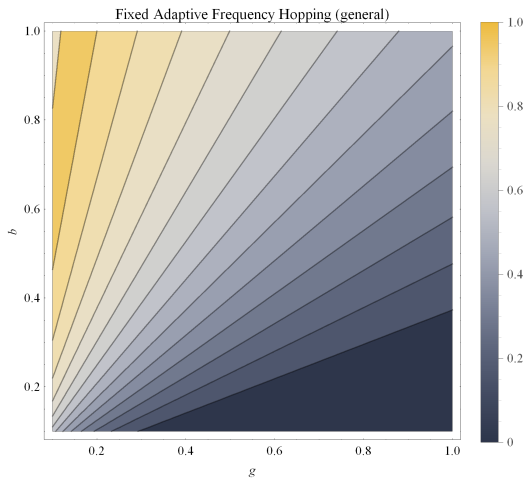


Fig. 3. Graph of the combined data rate and interference metric plotted vs time constants  $g$  and  $b$  for both the fixed adaptive frequency hopper. Compared with the random frequency hopper, it diminishes the range of almost no data and increases data rates across the board.

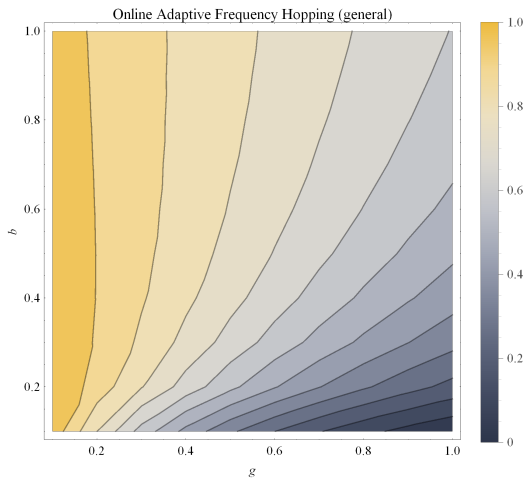


Fig. 4. Graph of the combined data rate and interference metric plotted vs time constants  $g$  and  $b$  for both the online adaptive frequency hopper. Note the curve in the structure of the graph. Additionally, note how whereas the other two protocols only achieved high data rates at high  $b$ , the online adaptive frequency hopper can maintain high data rates at low  $b$ .



## VII. PHYSICAL TESTS

### A. *Materials*

The physical experiment required four Universal Software Radio Peripheral (USRP) devices, an ethernet switch, a computer running Ubuntu 12.04, GNURadio software, and an implementation of random frequency hopping (Bluetooth), fixed adaptive frequency hopping (IEEE 802.22), and Online Adaptive Frequency Hopping (the new protocol) in Python.

### B. *Set up*

One pair of USRP devices was used to represent the primary users and one pair of USRP devices was used to represent the secondary user. The primary user pair represented all primary users on a given channel.<sup>4</sup> Each primary-user pair will had a radio designated as "transmitter," whose responsibility was to send the data and a radio designated as "receiver," whose responsibility was to monitor the channel for information from the primary. Likewise, the secondary user radio had both a "transmitter" and "receiver" node to handel its communications. This secondary's transmitter node additionally acted as a sensor, when appropriate in the protocol.

All of the devices were connected via coaxial cable. Each device was the connected to a computer running Ubuntu 12.04 and Python 2, while would control how the radio behaved. Additionally, three data files which contained the times for the primary user to send data were created.

### C. *Control*

For the control, the USRP representing the secondary user was powered off. Using one of the data files from the set up stage, the transmitter-node for the primary user send data at its specified times. The receiver-node primary user recorded the total amount of data received during this period. This was recorded and run an additional 9 times (for a total of 10) in order to both accurately determine the result and the random fluctuations due to the coaxial cable and other sources.

<sup>4</sup>To run the experiment with more channels, there needs to be an additional pair of primary users per channel.

#### *D. Experiment*

For the experiment, the USRP representing the secondary user was powered on. The primary user then used one of the data files to transmit data like in the control test. The secondary user was then activated and began behaving as specified by the protocol being tested (either random frequency hopping, fixed adaptive frequency hopping, or online adaptive frequency hopping). The receiver-node of both the primary and secondary users then recorded the amount of data received. The computer running the simulation also noted any clashes between the two. The experiment was then repeated for a total of 10 trials for each protocol and primary user data file.

#### *E. Transmission*

In order to differentiate between the signals sent from the primary user and those sent from the secondary user, the primary user sent packets that contain all 0's and the secondary user sent packets that contain all 1's. Both of these are unlikely to occur from random fluctuations and easy to be disrupted.

#### *F. Random Frequency Hopping*

As there is only one channel, the random frequency hopper tried to transmit all the time. This allows one to use the data rate of the random frequency hopper as a theoretical maximum for data rate of the secondary user.

#### *G. Fixed Adaptive Frequency Hopper*

The fixed adaptive frequency hopper sensed every thirty seconds and only sent data if the channel is available.

#### *H. Online Adaptive Frequency Hopper*

The online adaptive frequency hopper behaved the same as the fixed adaptive frequency hopper, except that the number of hops made by the radio varied.

## VIII. RESULTS

The protocol performed very well. In the theoretical analysis and simulations, the online adaptive frequency hopper saw 35% increase in the combined data rate and interference metric compared to random frequency hopping. The online adaptive frequency hopper also saw a 26%

increase in the combined metric when compared to the fixed sensing rate of 30 seconds specified in IEEE 802.22. In general, the protocol outperformed the fixed sensing rate radio when variations occurred in the environment (e.g. the cycle of usage in a day and the sudden spike in usage when there is an emergency). The online adaptive frequency hopper also outperformed the fixed version in times when the environment did match the environment. Additionally, the "double-back" method usually performed better than the "single-back" method of adaptive frequency hopping due to the reduction of sensing error.

## IX. CONCLUSION AND FUTURE WORK

The simulation of the various protocols proved successful. Given sufficient periods of time to run, the mathematics accurately predicted the results of the simulation. Both forms of adaptive frequency hopping performed better than randomly selecting channels. Additionally, the online method of frequency hopping performed better than selecting a fixed rate at which to sense as it could optimize the sensing rate for the environment. Being able to change sensing rates allowed the online adaptive frequency hopper to take advantage of the random fluctuations in system. However, the greatest strength of the new protocol was its ability to achieve very high data rates when the environment was not changing while maintaining low interference when channel availability was limited. As seen in the simulations, there were specific regions in  $b$  and  $g$ -space where the protocol was unable to model the environment. However, these areas only occurred when  $b$  was much smaller than  $g$ , meaning the optimal data rate was close to zero.

While the protocol performed very well compared to its predecessors, there are still several places to improved. First and foremost, more research can be done with the confusion matrix to determine the conditions which minimize sensing error without eliminating all possible channels. Additionally, the protocol can be tested on multiple channels with additional USRP's to further optimize for conditions that the simulated model did not test.

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## X. APPENDIX - CALCULATIONS

The following calculations were used to evaluate data rate and interference for all protocols. In all calculations  $D$  represented data rate,  $I$  represented interference,  $b$  represented the time constant for channel unavailability,  $g$  represented the time constant for channel availability,  $N$  represented the number of channels that the radio could use,  $d$  represented the percent of time that any channels were calculated to be available,  $R_n$  represented the probability that an available channel was incorrectly judged to be unavailable,  $R_p$  represented the probability that an unavailable channel was incorrectly judged to be available,  $E_t$  represented the expected number of correct positive detections,  $E_f$  represented the expected number of false positive detections, and  $H$  represented the number of hops in a given frame (the inverse of sensing frequency). The calculations for both "single-back" and "double back" represent a fixed adaptive frequency hopper, but can derive the calculations for an online adaptive frequency hopper by taking the local maximum with respect to  $H$ .

### A. Random

$$D_R = \frac{b}{b+g} = p \quad (11)$$

$$I_R = \frac{g}{g+b} = 1 - p \quad (12)$$

### B. Single-Back

$$E_t = N \cdot p \cdot (1 - R_n) \quad (13)$$

$$E_f = N \cdot (1 - p) \cdot R_p \quad (14)$$

$$d = 1 - \left(1 - \frac{E_t + E_f}{N}\right)^N \quad (15)$$

$$D_B = \frac{d \cdot (E_t \int_0^{Ht_c} \pi(t) dt + E_f \int_0^{Ht_c} \phi(t) dt)}{(Ht_c + t_s)(E_t + E_f)}$$

$$I_B = \frac{d \cdot (E_t \int_0^{Ht_c} 1 - \pi(t) dt + E_f \int_0^{Ht_s} 1 - \phi(t) dt)}{(Ht_c + t_s)(E_t + E_f)}$$

*C. Double Back*

$$E_t = N \cdot p \cdot (1 - R_n) \cdot (\pi(t)(1 - R_n) + (1 - \pi(t))R_p) \quad (16)$$

$$E_f = N \cdot (1 - p) \cdot R_p \cdot (\phi(t)(1 - R_n) + (1 - \phi(t))R_p) \quad (17)$$

$$d = 1 - \left(1 - \frac{E_t + E_f}{N}\right)^N \quad (18)$$

$$D_D = \frac{d \cdot (E_t \int_0^{Ht_c} \pi(t) dt + E_f \int_0^{Ht_c} \phi(t) dt)}{(Ht_c + t_s)(E_t + E_f)}$$

$$I_D = \frac{d \cdot (E_t \int_0^{Ht_c} 1 - \pi(t) dt + E_f \int_0^{Ht_s} 1 - \phi(t) dt)}{(Ht_c + t_s)(E_t + E_f)}$$

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