Automated Bayesian Estimation of Quantum Error Models

William S. Moses (wmoses@mit.edu), Costin Iancu (cciancu@lbl.gov), Bert de Jong (wadejong@lbl.gov)

MIT CSAIL, Lawrence Berkeley National Lab

1) Describe error model as replacement rules for circuit elements as probability distributions over outcomes using Gen [1].

```
# Specifying a noisy rotation model of a U3 gate
@gen function errorRx(θ, stddev)
    return Rx(@trace(normal(θ, stddev)))
end

# Specifying an error model of a Hadamard gate
@gen function errorH(thres)
    return (@trace(uniform(0, 1)) < thres) ? H() : I()
end
```

2) Describe and simulate the circuit being run using Yao.jl [2]. Can also import/call QisKit [3].

```
# Sample circuit, simply executing a Hadamard gate
@gen function mycircuit(nsamples)
    thres = @trace(uniform(0, 1), :thres)
    circuit = errorH(thres)
    probs = Yao.probs(circuit)
    @trace(multinomial(probs, nsamples), :counts)
end
```

3) Assert that we measured the given data and find the most probable error parameters by running importance sampling then gradient ascent.

```
# Assert our measurements are given by the data
observations = choicemap(:counts => data)

# Find the most probable parameter settings
(trace, _) = importance_resampling(circuit, (size(data,1),),
                                   observations, 400);
trace = adam_optimize(trace, 1000)
println(get_choices(trace)[:thres])
```

Figure 1. We use probabilistic programming and circuit rewrite rules to automatically derive error models from data.

Desired Properties of Quantum Error Tools

- Learn an error model given ***pre-existing training data*** of circuits that the tool does not choose.
- Derive ***custom error models***.
- Explain ***why*** the learned error model was ***chosen***.
- Explain ***how*** the derived error model ***relates*** to underlying hardware / physical phenomena.

	Existing Data	Custom Models	Why Model	Errors Relate
OS[4]	✗	✓	✗	✓
SLYB[5]	✓	✗	✗	✗
RB[6]	✗	✓	✗	?
HFW[7]	✗	✗	✗	?
Us	✓	✓	✓	✓

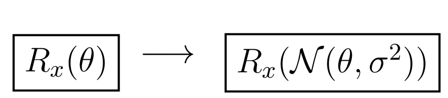
Figure 2. Generally prior tools suffer from one of three issues: they require running particular circuits and thus don't work with existing data (OS[4], RB[6], HFW[7]), they only work with limited error models (SLYB[5], HFW[7]).

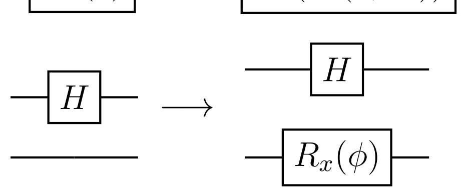
Possible Uses

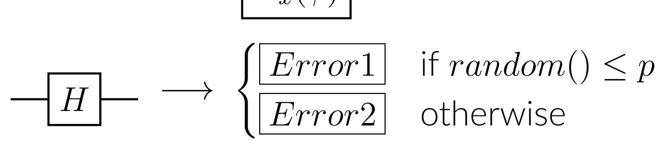
- Can predict how well your circuit will run in advanced
- Can select better circuit/layout that minimizes errors
- Can validate more sophisticated error model

Building Quantum Error Models

Represent arbitrary error models as replacements for components of the circuit by a parameterized probabilistic circuit. Generality arises from composition of error models.

- Noisy Rotation: 

- Gate Leakage: 

- Model Selection: 

Bayesian Error Learning

- Derive the most likely error parameter θ by applying Bayes rule to the calculation of the circuit's end state

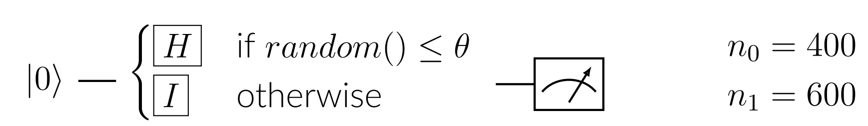


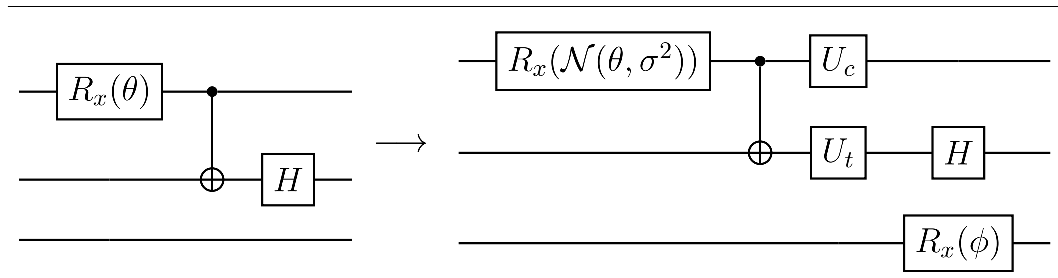
Figure 3. Circuit whose Hadamard gate works only some fraction θ of the time. Actual measured counts are 400 and 600 for $|0\rangle$ and $|1\rangle$ respectively.

$$p(n_0|\theta) = \binom{n_0+n_1}{n_0}\left(\frac{\theta}{2}+(1-\theta)\right)^{n_0}\left(\frac{\theta}{2}\right)^{n_1}$$

$$p(\theta|n_0) = \frac{p(data|\theta)p(\theta)}{p(data)} \propto p(data|\theta)p(\theta)$$

$$\theta^* = \arg\max_\theta p(\theta|n_0) = 0.8$$

Automatic Workflow



- Apply rewrite rules from error model to create parameterized circuit on the right
- Manual Bayesian analysis intractable
- Use Workflow in Figure 1 to derive error model.

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Validation & Conclusion

- Run Qiskit's simulator with random noise models and rederive the parameters of said noise model.
- Run a circuit on IBM quantum computer, train the error model on a subset of the data, and see how it generalizes to the rest of the dataset.

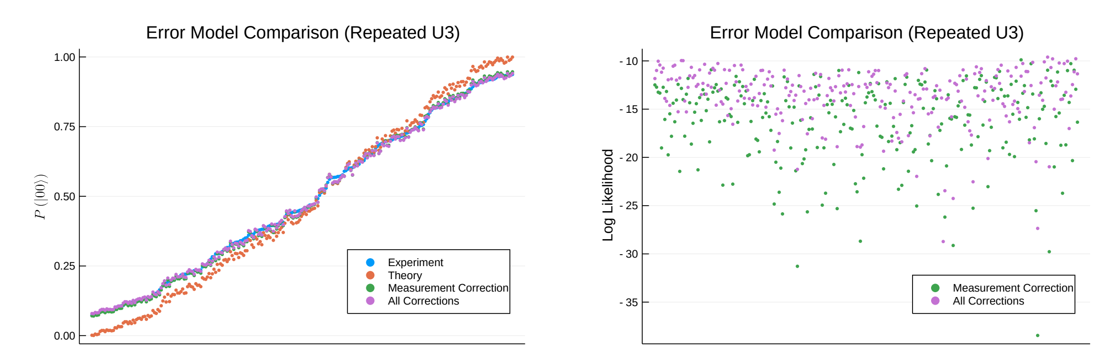

Figure 4. Left: the probability of measuring $|0\rangle$ after running two $U3$ gates, as predicted by theory (orange), accounting for readout error correction (green), accounting for over rotation and gate bias (orange), and in experimental data (blue). Right: the log likelihood that learned error models match experimental data from the right.

- Ran 512 random circuits
- Given more powerful models (first only readout error, then overrotation and bias), closer match data without overfitting
- Workflow can both derive accurate error models from experimental data as well as accurately simulate further data

References

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