

# Polygeist: Raising C to Polyhedral MLIR



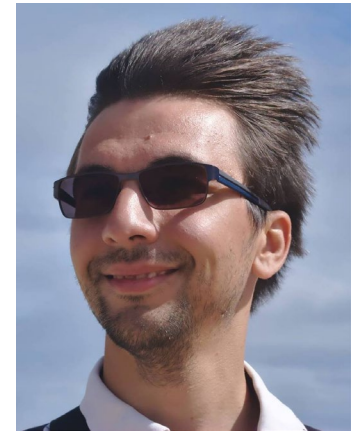
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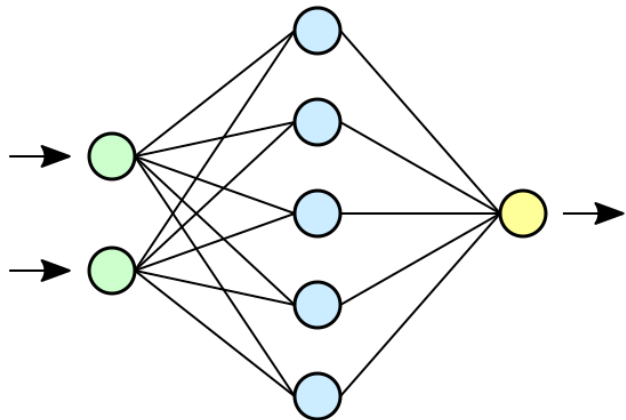


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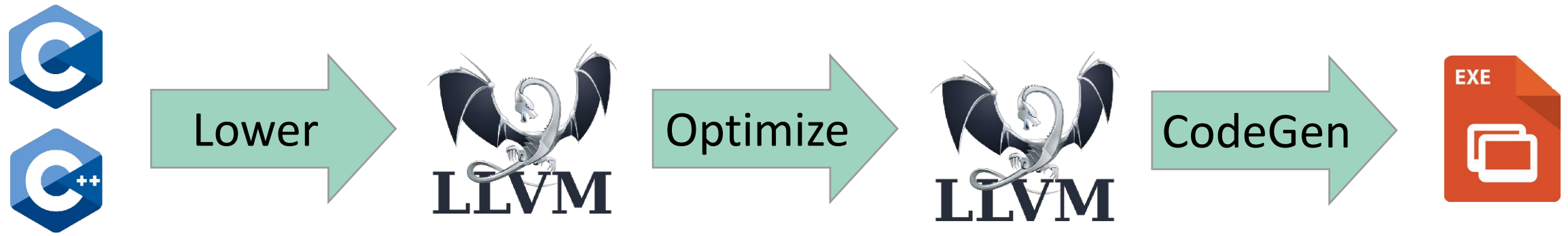
PACT  
September 2021

# The Polyhedral Model

- Makes it easy to analyze and specify program transformations best exploit the available hardware
  - Loop restructuring for spatial/temporal locality, automatic parallelization, etc.
- One of the best frameworks for optimizing compute-intensive programs like machine learning kernels or scientific simulations as well as for programming accelerators.



# Polyhedral Compilation Today



Source-Based  
Transformation

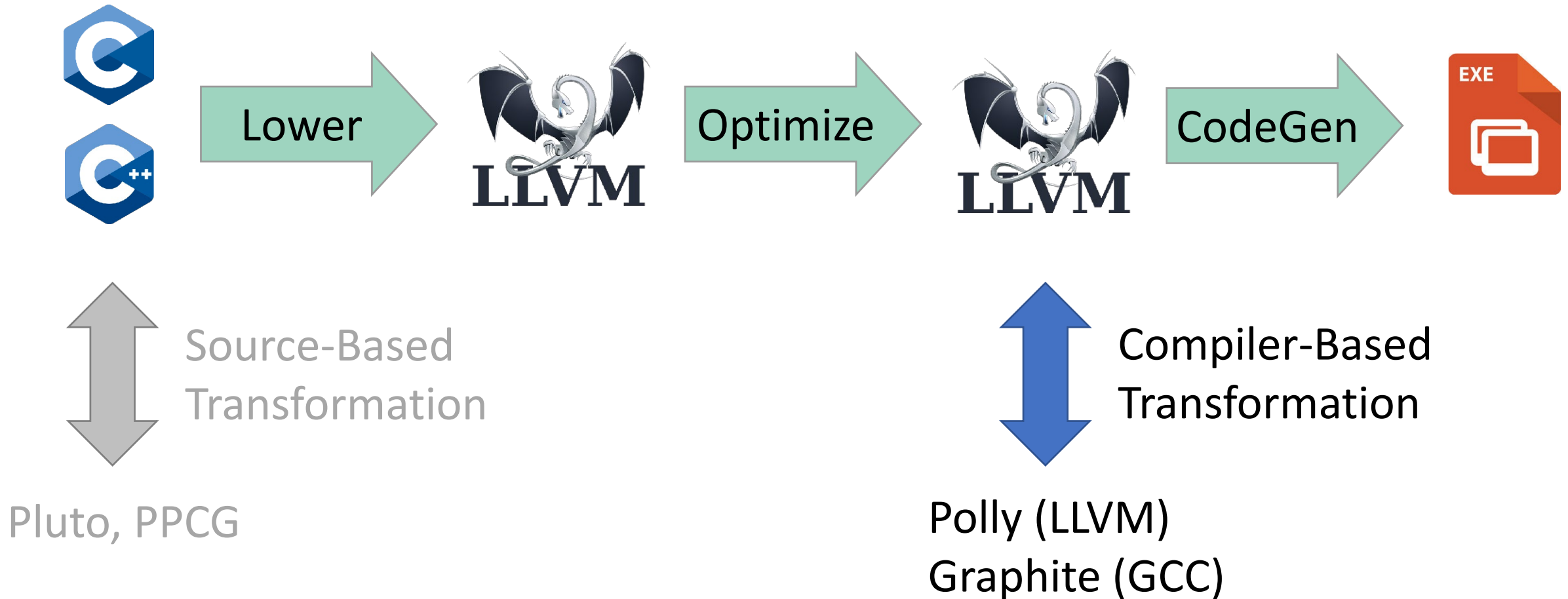
Pluto, PPCG



Compiler-Based  
Transformation

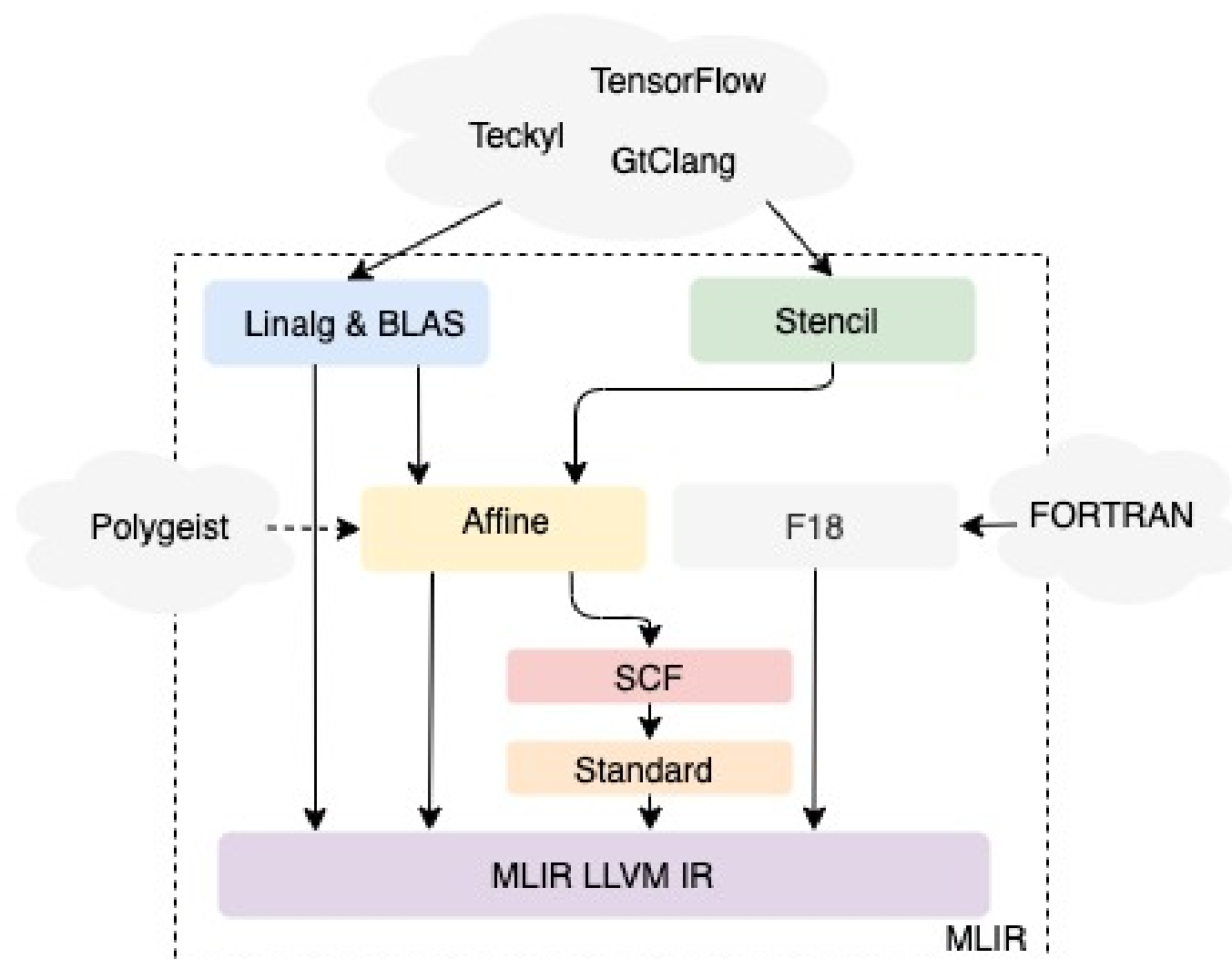
Polly (LLVM)  
Graphite (GCC)

# Polyhedral Compilation Today





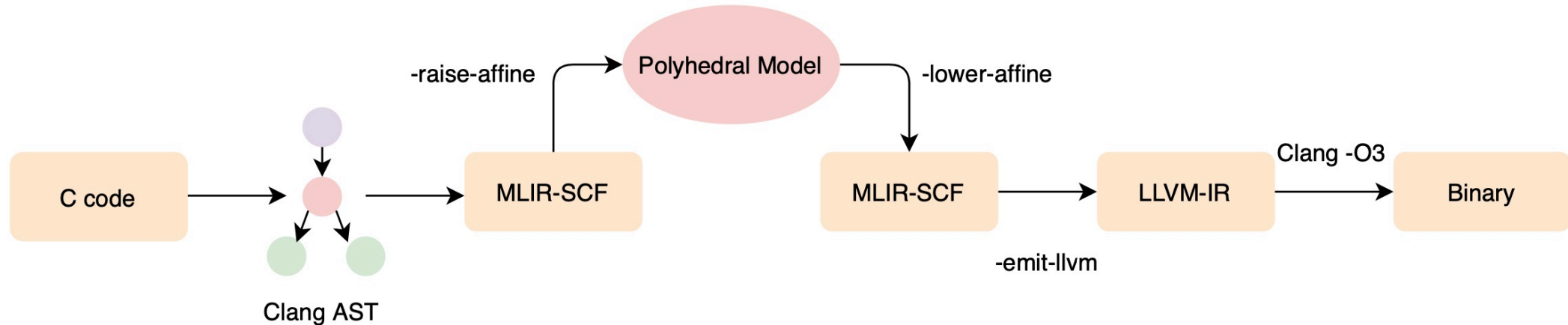
# The MLIR Framework



# Polygeist

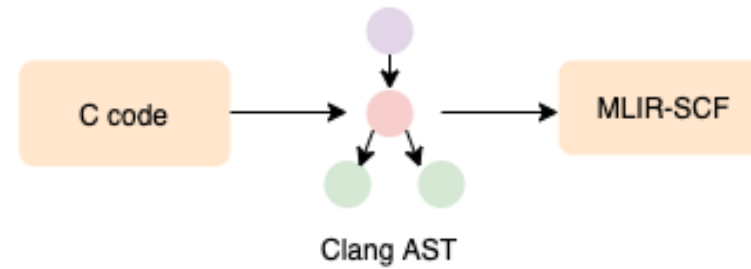
- An MLIR-based end-to-end polyhedral compilation flow
- Keeps the best parts of high-level abstractions and low-level optimizations by directly lowering to and optimizing MLIR.
- Multiple abstraction levels allows new polyhedral optimization opportunities and simplifies the implementation of others
- State-of-the-art performance when compared with existing source and compiler-based tools.

# The Polygeist Compilation Flow



- Generic C or C++ frontend that generates "standard" MLIR
- Raising transformations for transforming "standard" MLIR to polyhedral MLIR (Affine)
- Embedding of existing polyhedral tools (Pluto, CLooG) into MLIR
- Novel transformations (statement splitting, reduction detection) that rely on high-level compiler representation
- End-to-end evaluation of standard polyhedral benchmarks (Polybench)

# Polygeist Frontend



- Ingests Clang AST to simplify parsing, semantic analysis, linkage, etc.
- Each C/C++ type is defined to have a corresponding MLIR. Pointers, arrays, and some structs can use MLIR's structured pointer or memref type, preserving sizes and multi-dimensional indexing.
  - `int[12][30] => memref<12x30xi32>`
  - `float*` `=> memref<?xf32>`
- Allocation and deallocation instructions converted to memref alloc/dealloc.
- Control flow and loops are directly lowered to structured MLIR-equivalent operations.
- Supports advanced C++ features like templates and constructors.

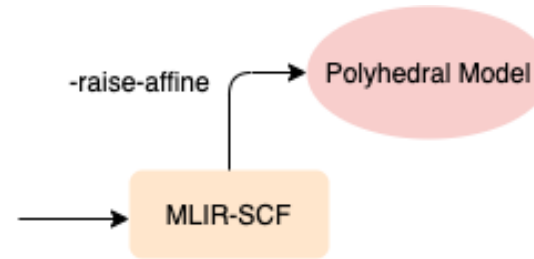


# Simplified Frontend Example

```
void set(int *arr, int val) {  
    for(int i=0; i<10; i++){  
        arr[2*i] = val;  
    }  
}
```

```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
    %c0 = constant 0 : index  
    %0 = alloca() : memref<1xmemref<?xi32>>  
    store %arg0, %0[%c0] : memref<1xmemref<?xi32>>  
    %1 = alloca() : memref<1xi32>  
    store %arg1, %1[%c0] : memref<1xi32>  
    %c0_i32 = constant 0 : i32  
    %c2_i32 = constant 2 : i32  
    %c10_i32 = constant 10 : i32  
    %2 = index_cast %c10_i32 : i32 to index  
    scf.for %arg2 = %c0_i32 to %2 {  
        %3 = index_cast %arg2 : index to i32  
        %4 = alloca() : memref<1xi32>  
        store %3, %4[%c0] : memref<1xi32>  
        %5 = load %0[%c0] : memref<1xmemref<?xi32>>  
        %6 = load %4[%c0] : memref<1xi32>  
        %7 = muli %c2_i32, %6 : i32  
        %8 = index_cast %7 : i32 to index  
        %9 = load %1[%c0] : memref<1xi32>  
        store %9, %5[%8] : memref<?xi32>  
    }  
    return  
}
```

# Polygeist Raising



- Local variables eliminated by new MLIR mem2reg pass
- Canonicalizations run to simplify the code, including while => for
- Raising an operation (for, if, load, store) to its affine-variant:
  - Detect if index calculation is a valid affine expression
  - Progressively fold index calculation into a new replacement affine operation

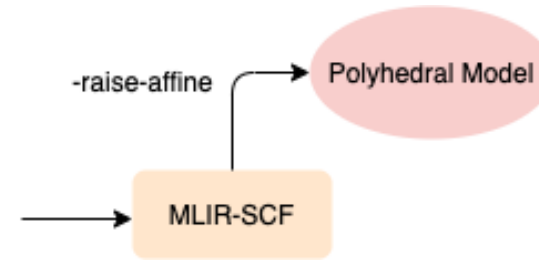
```
%off = muli %c2, %idx : index  
store %val, %ptr[%off] : memref<?xi32>
```



```
affine.store %val, %ptr[2 * %idx] : memref<?xi32>
```

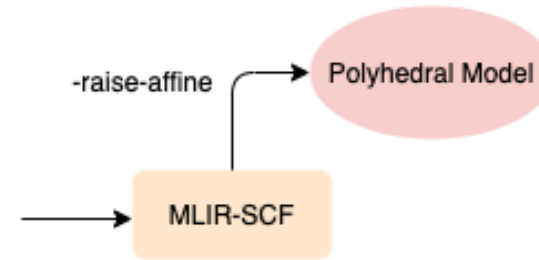
# Polygeist Raising

```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  %c0 = constant 0 : index  
  %0 = alloca() : memref<1xmemref<?xi32>>  
  store %arg0, %0[%c0] : memref<1xmemref<?xi32>>  
  %1 = alloca() : memref<1xi32>  
  store %arg1, %1[%c0] : memref<1xi32>  
  %c0_i32 = constant 0 : i32  
  %c10_i32 = constant 10 : i32  
  %2 = index_cast %c10_i32 : i32 to index  
  scf.for %arg2 = %c0_i32 to %2 {  
    %3 = index_cast %arg2 : index to i32  
    %4 = alloca() : memref<1xi32>  
    store %3, %4[%c0] : memref<1xi32>  
    %5 = load %0[%c0] : memref<1xmemref<?xi32>>  
    %c2_i32 = constant 2 : i32  
    %6 = load %4[%c0] : memref<1xi32>  
    %7 = muli %c2_i32, %6 : i32  
    %8 = index_cast %7 : i32 to index  
    %9 = load %1[%c0] : memref<1xi32>  
    store %9, %5[%8] : memref<?xi32>  
  }  
  return  
}
```



# Polygeist Raising

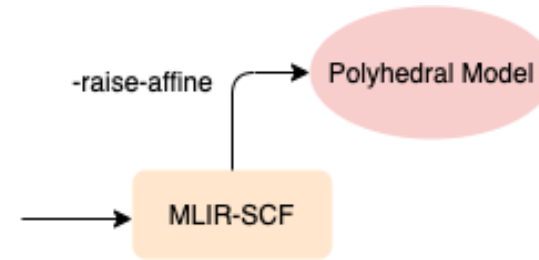
```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  %c0 = constant 0 : index  
  
  %c0_i32 = constant 0 : i32  
  %c10_i32 = constant 10 : i32  
  %2 = index_cast %c10_i32 : i32 to index  
  scf.for %arg2 = %c0_i32 to %2 {  
    %3 = index_cast %arg2 : index to i32  
  
    %c2_i32 = constant 2 : i32  
  
    %7 = muli %c2_i32, %3 : i32  
    %8 = index_cast %7 : i32 to index  
  
    store %arg1, %arg0[%8] : memref<?xi32>  
  }  
  return  
}
```



## 1. Mem2Reg

# Polygeist Raising

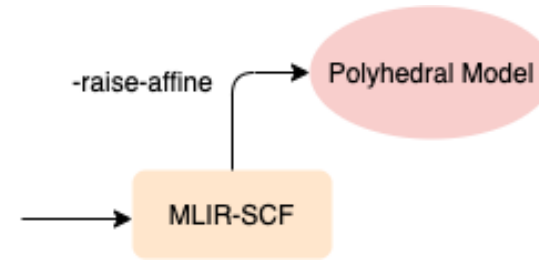
```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  %c0 = constant 0 : index  
  %c2 = constant 2 : i32  
  %c10 = constant 10 : i32  
  
  scf.for %arg2 = %c0 to %c10 {  
  
    %7 = muli %c2_i32, %arg2 : index  
  
    store %arg1, %arg0[%7] : memref<?xi32>  
  }  
  return  
}
```



1. Mem2Reg
2. Canonicalize

# Polygeist Raising

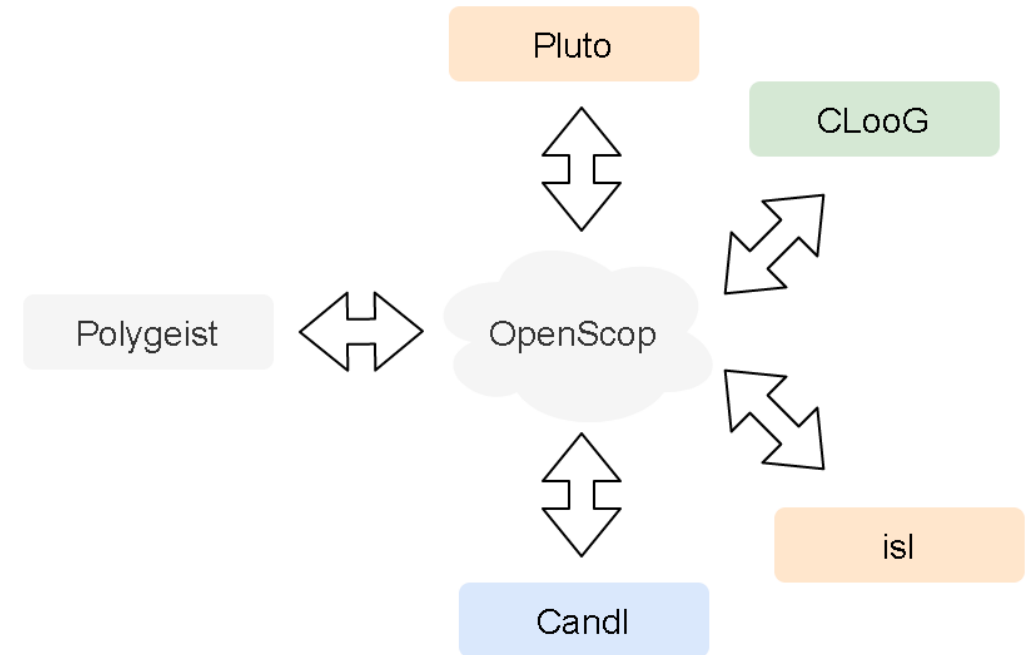
```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  
    affine.for %arg2 = 0 to 10 {  
  
        affine.store %arg1, %arg0 [2 * %arg2] :  
                                memref<?xi32>  
    }  
    return  
}
```



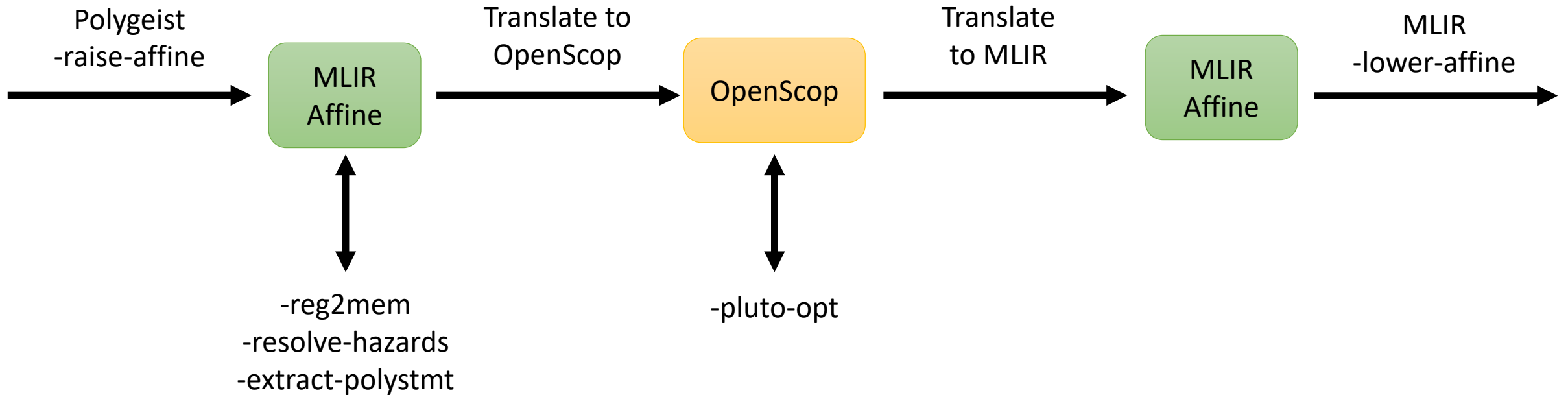
1. Mem2Reg
2. Canonicalize
3. Raise to Affine

# Connecting MLIR to Polyhedral Tools

- Polygeist produces MLIR **Affine**
- MLIR **Affine**  $\leftrightarrow$  polyhedral model
- Existing tools don't take **Affine**
- **OpenScop** is a target representation
- How to translate **Affine**  $\leftrightarrow$  **OpenScop**?



# Polyhedral Optimization Pipeline



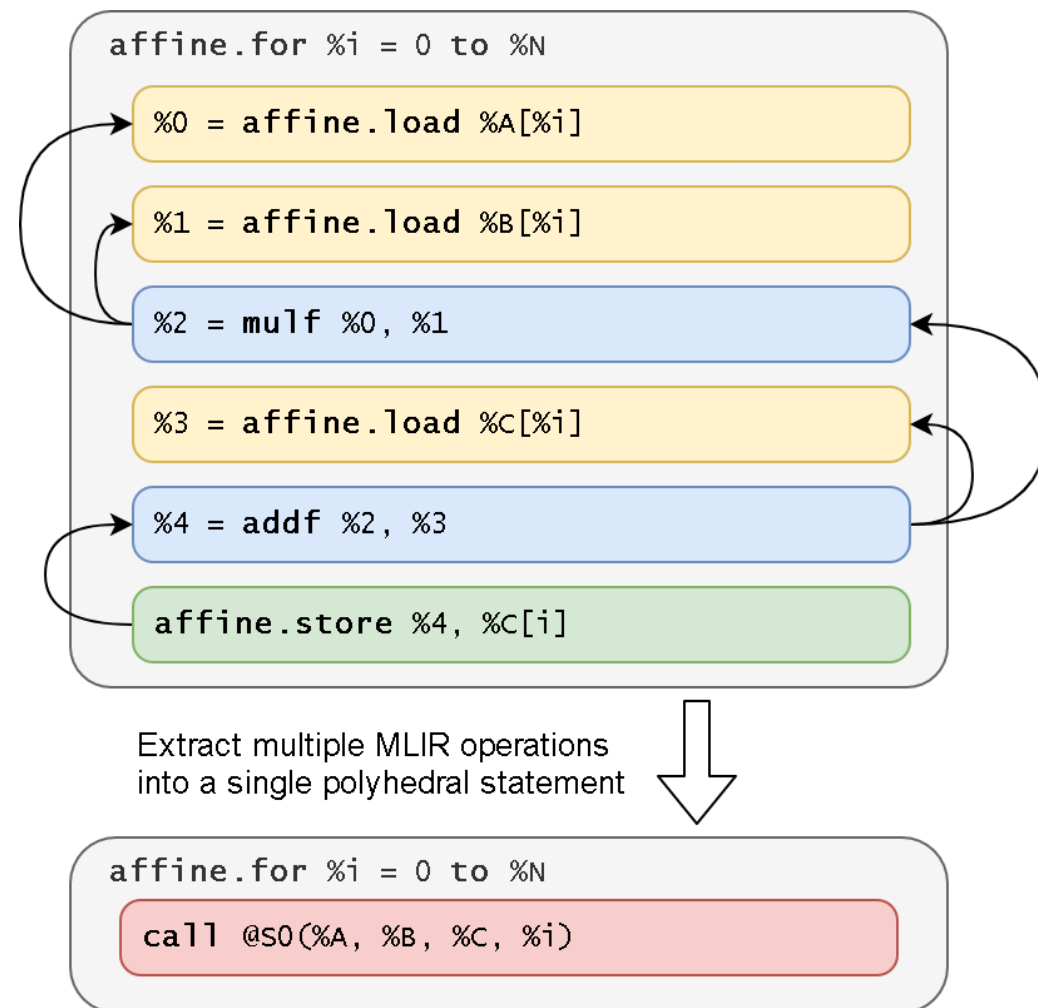


# Polyhedral Statement

- **OpenScop** expects **C-like** statements:

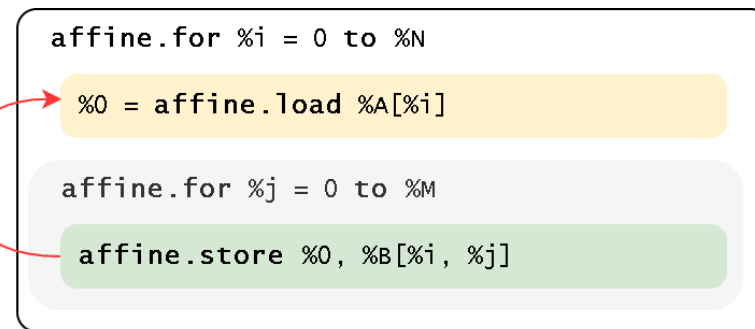
```
C[i][j] += A[i][k] * B[k][j]
```

- MLIR **Affine** is at a lower level.
- To match C-like statements:
  - **Extract** 1 MLIR memory write
  - **Traverse** SSA use-def chains
  - **Gather** until loads or symbols

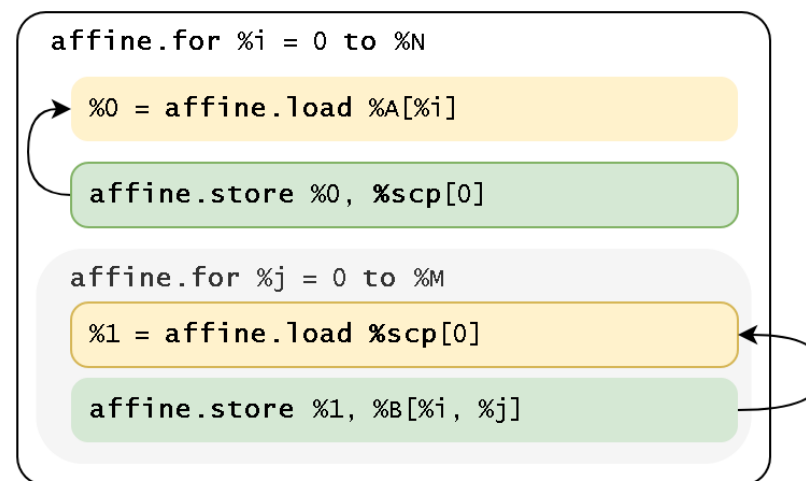


# Region-Spanning Problem

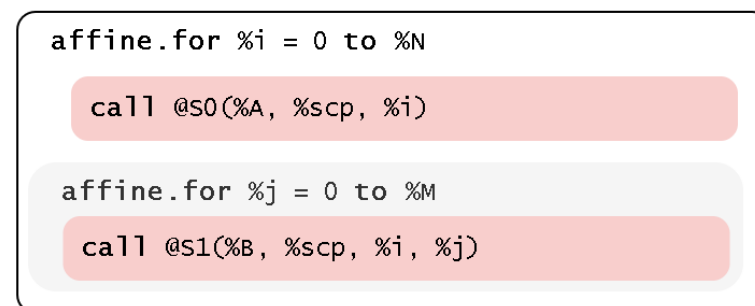
- A use-def chain spans across loops
- Statement nesting is **ambiguous**
- MLIR reconstruction is **difficult**
- Reg2mem pass: insert a **scratchpad** for each region-spanning use-def chain



The **reg2mem** pass that  
inserts a scratchpad

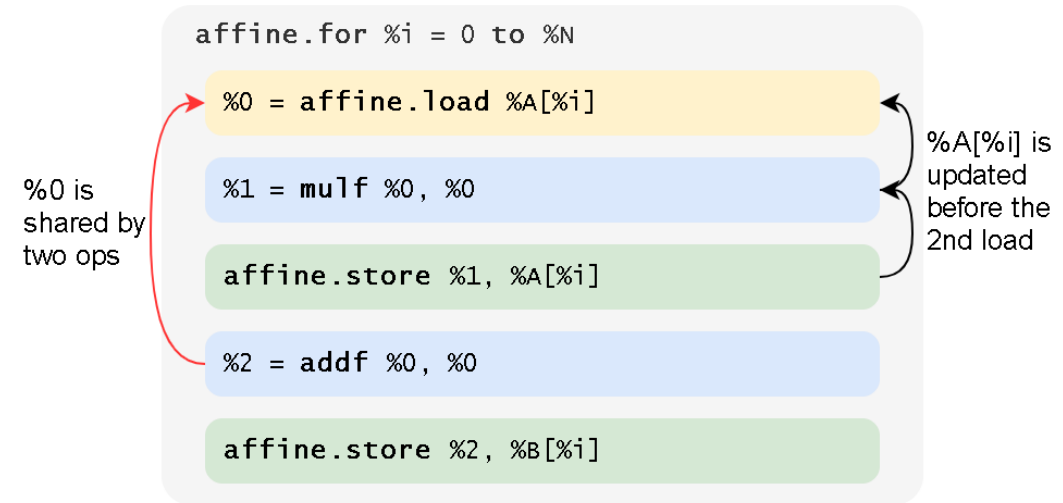


Extract polyhedral  
statements

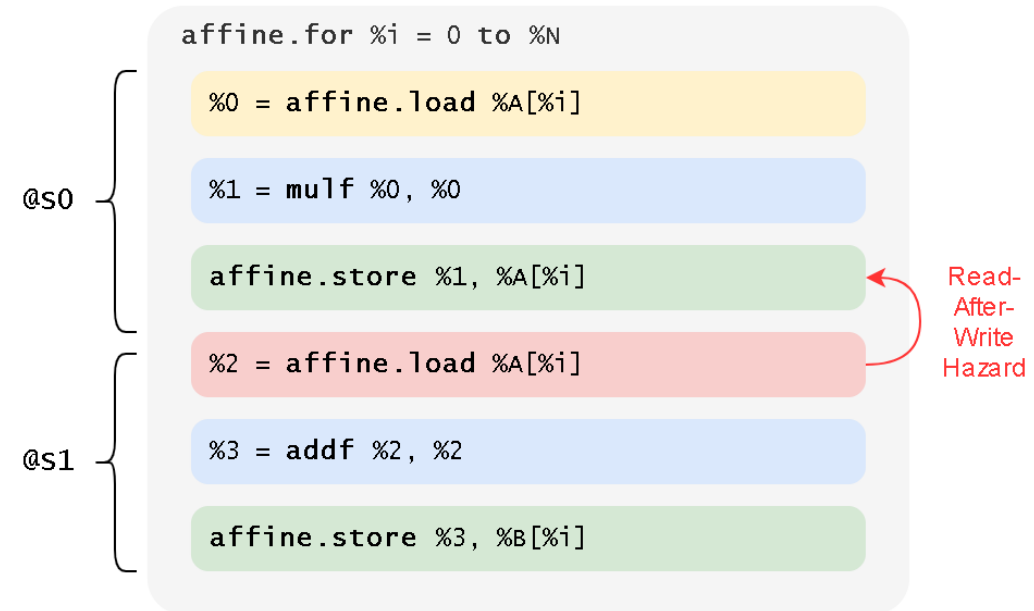


# Avoid RAW Hazard

- Two stores **share** the same load
- 1st store **overwrite** the address
- Detected by **value analysis**
- **Solution:** insert scratchpads



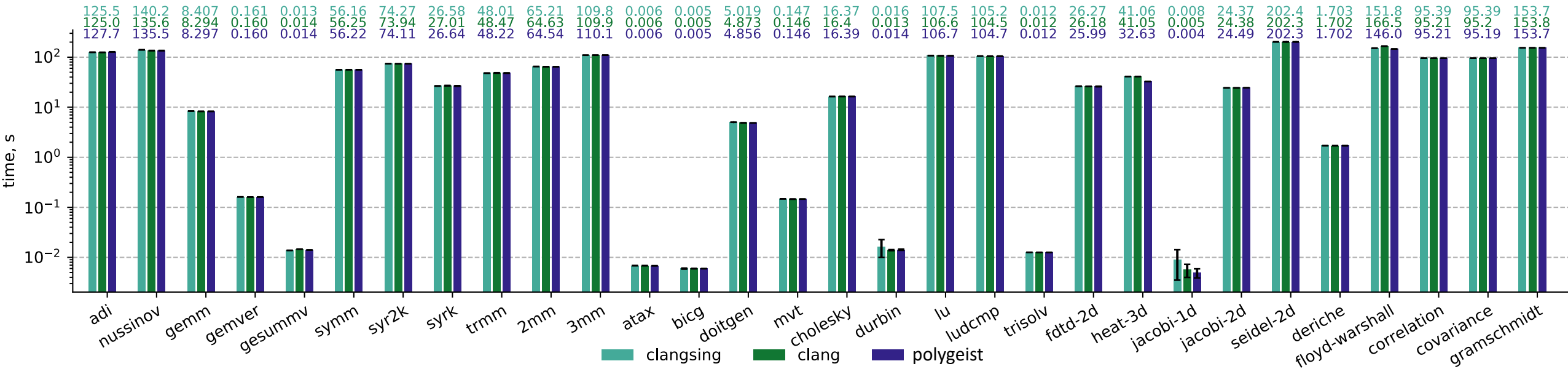
Duplicating shared operation  
may introduce hazards



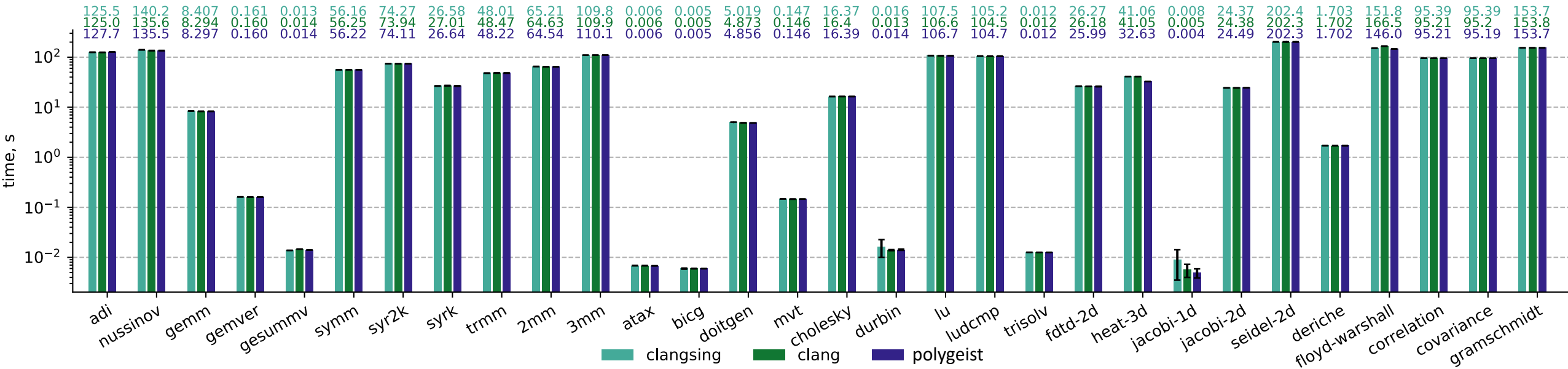
# Evaluation

- Compare Polygeist frontend with Clang to establish a fair baseline
- Compare Polygeist polyhedral optimization with Pluto (source-based) and Polly (compiler-based)
- Novel optimizations

# Serial Non-Polyhedral Comparison

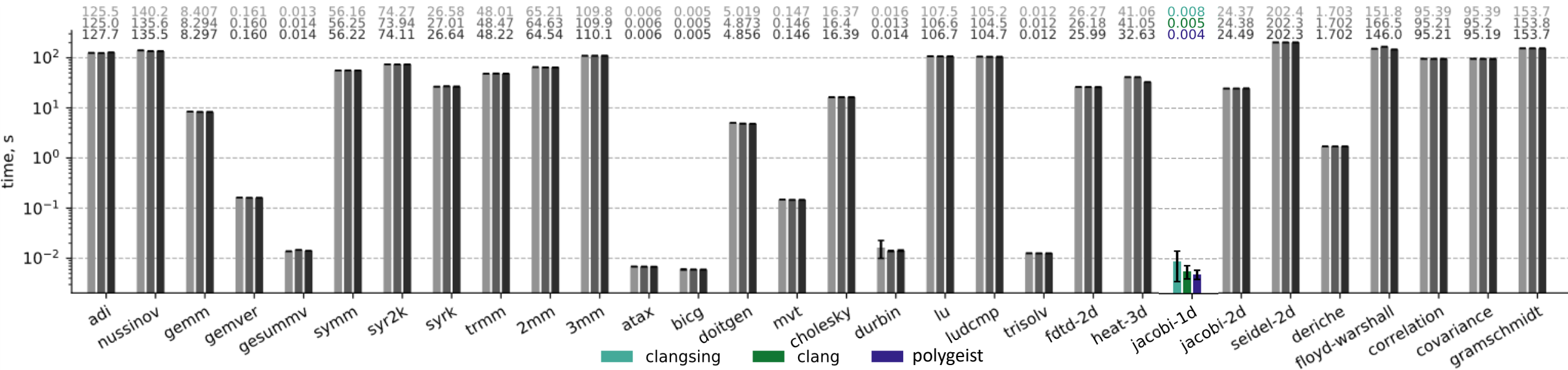


# Serial Non-Polyhedral Comparison



Frontend within 0.32% of  
“standard” frontend

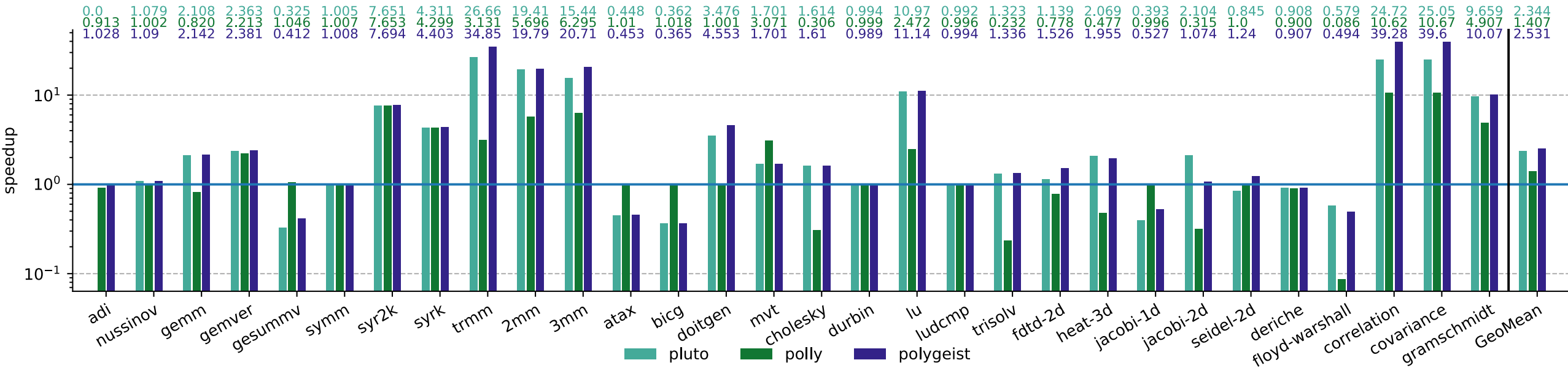
# Serial Non-Polyhedral Comparison



Frontend within 0.32% of  
“standard” frontend

Remaining gap attributed  
to small tests where minor  
assembly differences  
matter

# Sequential Polyhedral Comparison



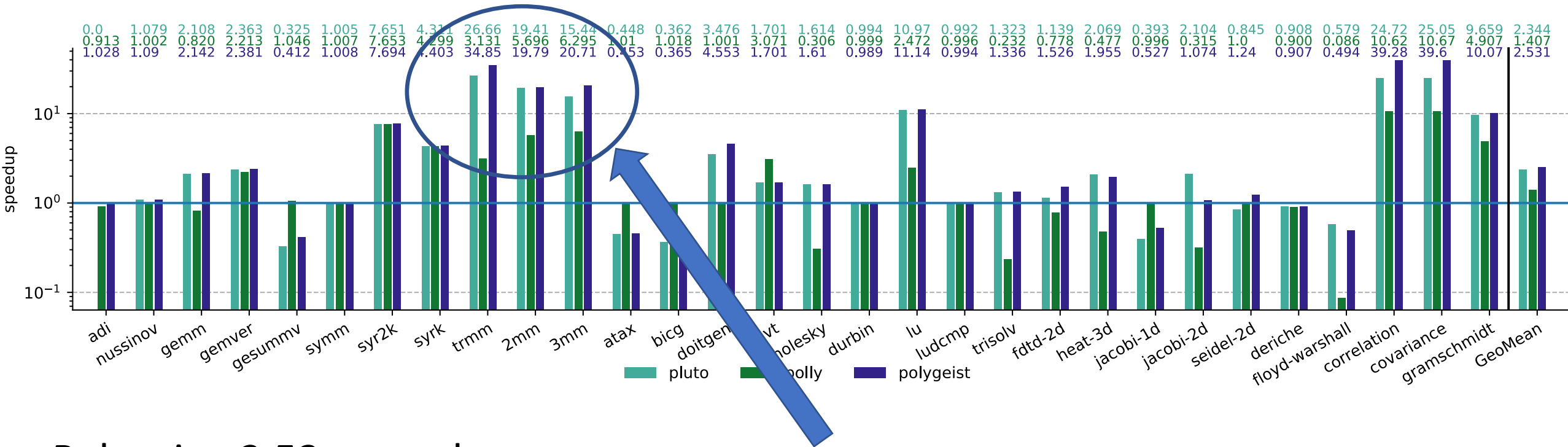
Polygeist: 2.53x speedup

Pluto: 2.34x speedup

Polly: 1.41x speedup



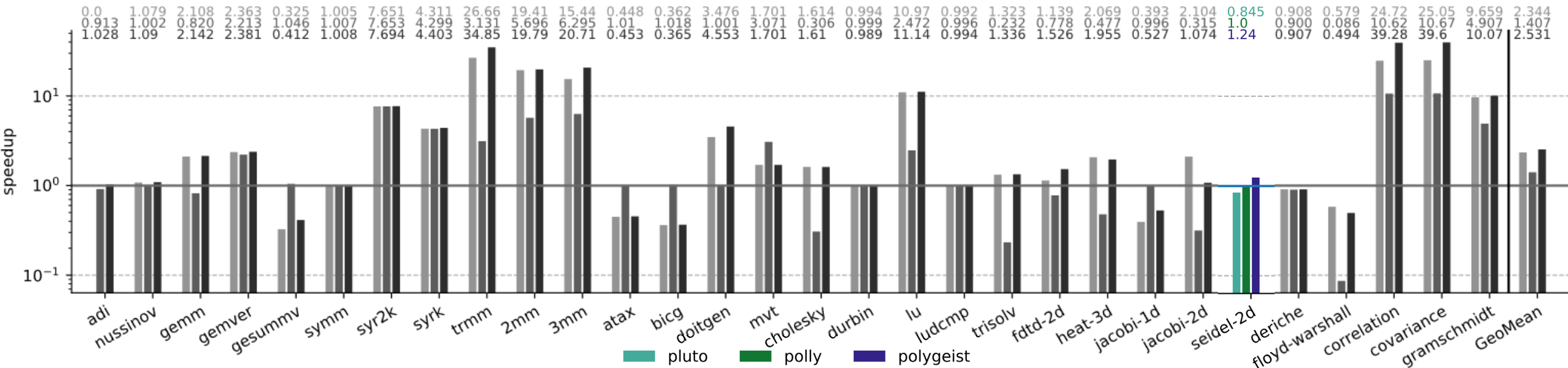
# Sequential Polyhedral Comparison



Polygeist: 2.53x speedup  
Pluto: 2.34x speedup  
Polly: 1.41x speedup

Big gaps come from  
different schedules

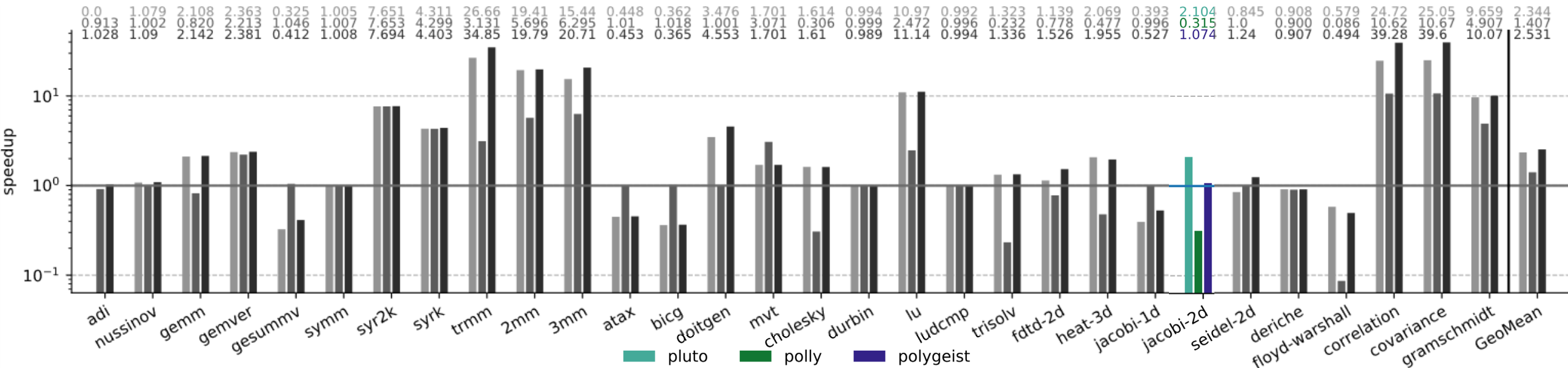
# Sequential Polyhedral Comparison



Polygeist: 2.53x speedup  
 Pluto: 2.34x speedup  
 Polly: 1.41x speedup

24% Polygeist speedup  
 comes from better integer  
 operations

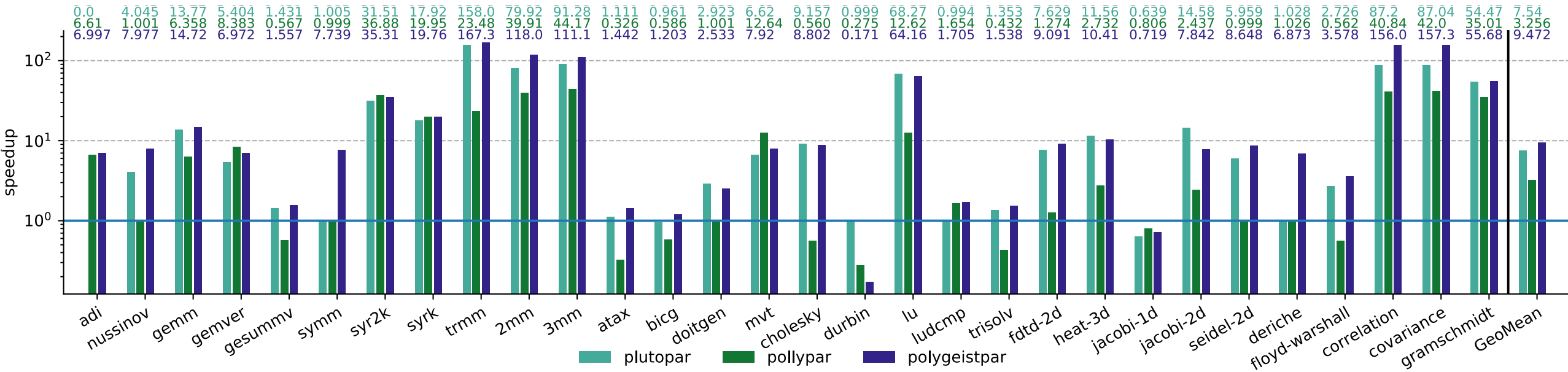
# Sequential Polyhedral Comparison



Polygeist: 2.53x speedup  
 Pluto: 2.34x speedup  
 Polly: 1.41x speedup

Polygeist slower due to  
 smaller branch-  
 simplification timeout

# Parallel Polyhedral Comparison

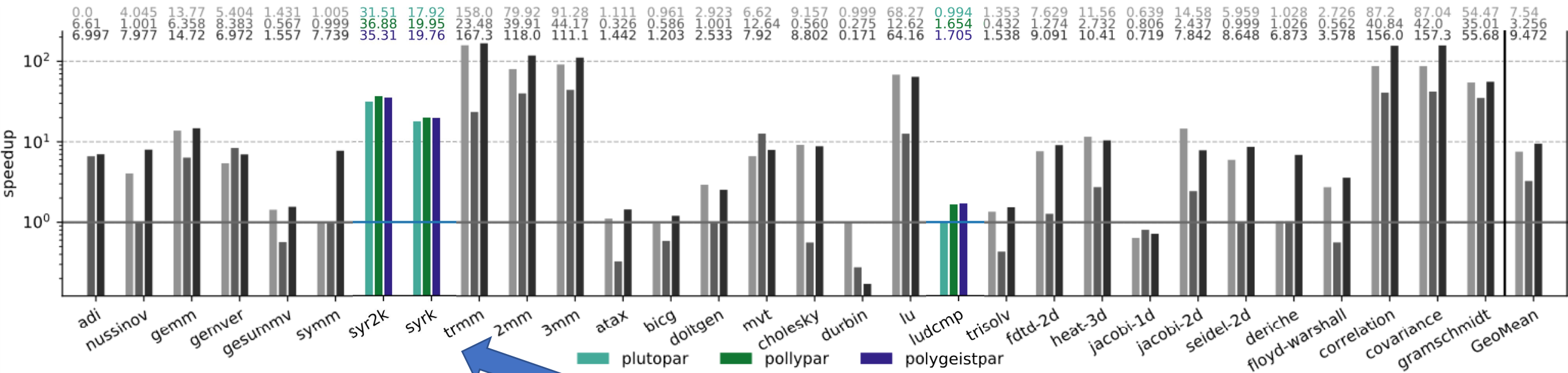


Polygeist: 9.47x speedup

Pluto: 7.54x speedup

Polly: 3.26x speedup

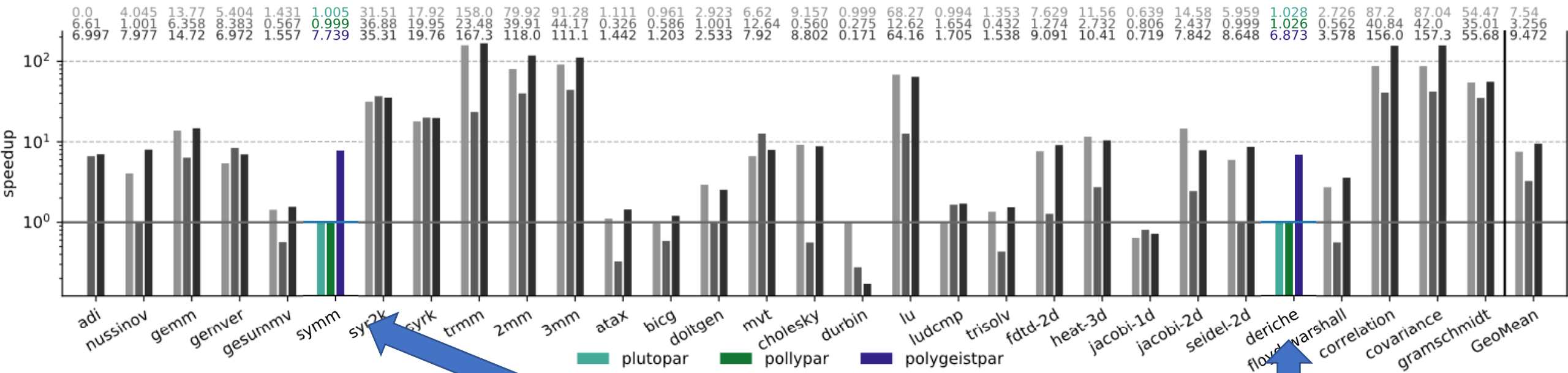
# Parallel Polyhedral Comparison



Polygeist: 9.47x speedup  
Pluto: 7.54x speedup  
Polly: 3.26x speedup

Polygeist and Polly benefit  
from SSA optimizations

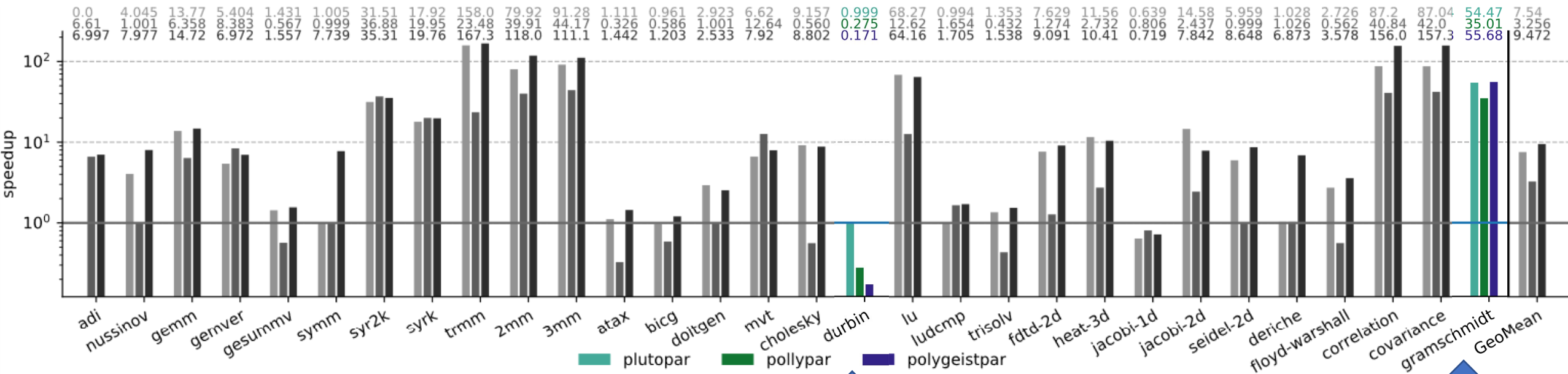
# Parallel Polyhedral Comparison



Polygeist: 9.47x speedup  
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 Polly: 3.26x speedup

Polygeist can remove loop-carried dependency and parallelize when others cannot

# Parallel Polyhedral Comparison



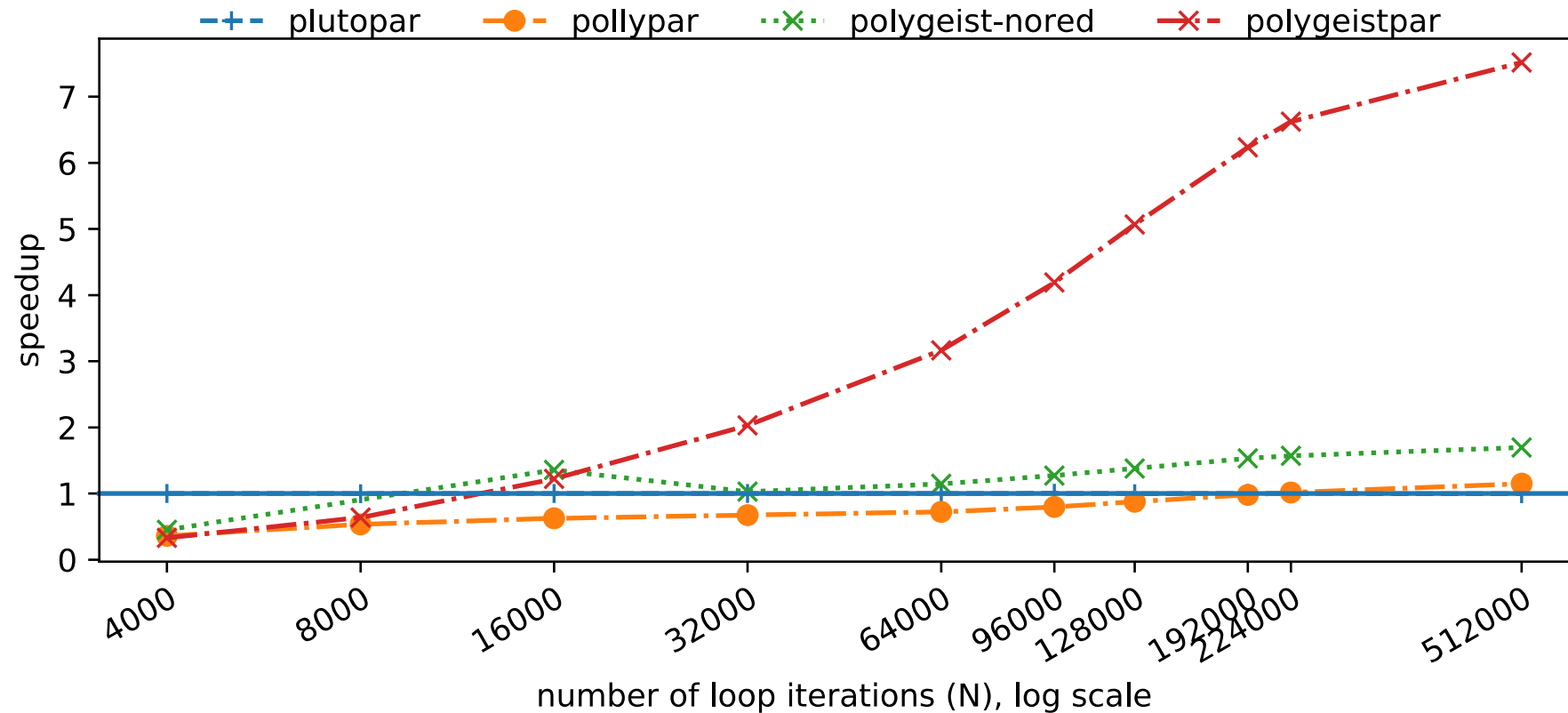
Polygeist: 9.47x speedup

Pluto: 7.54x speedup

Polly: 3.26x speedup

Polygeist can detect parallel reductions

# Parallel Reduction Detection (durbin)





# Statement Splitting

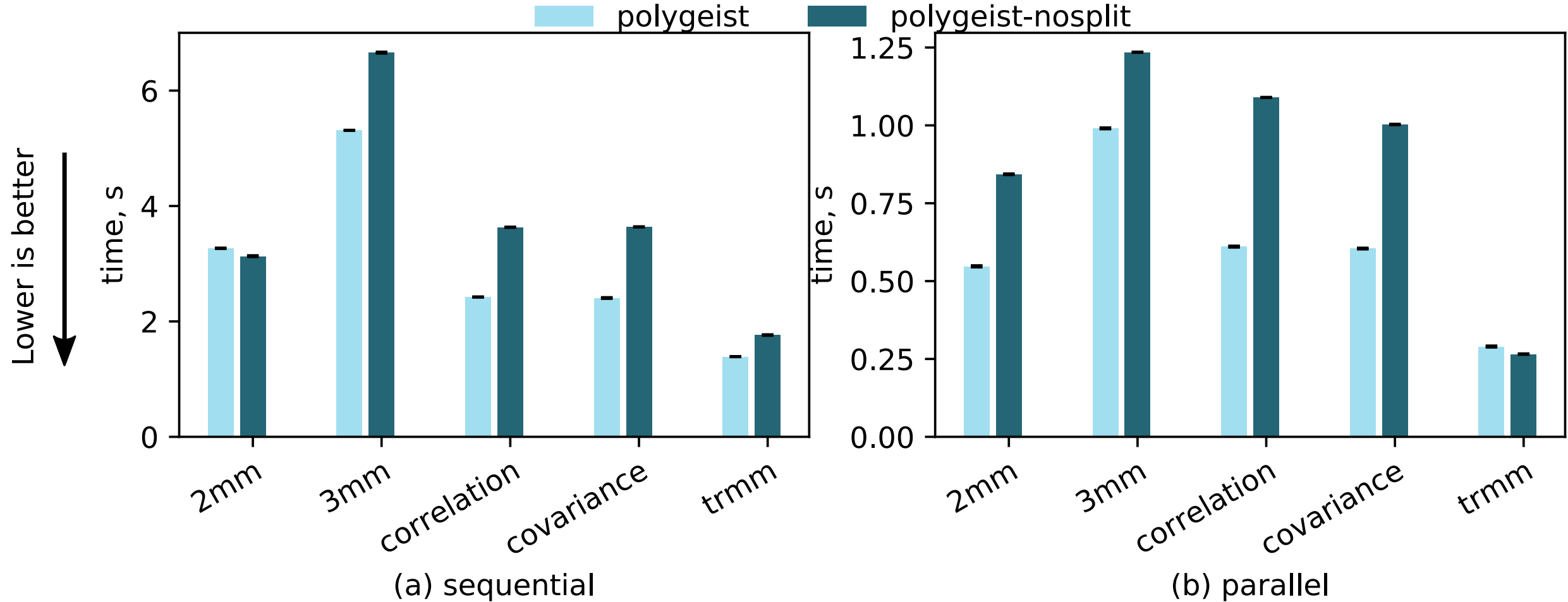
- We don't **need** to reconstruct the **original** C statements from Affine
- We can **split** statements by inserting **scratchpads**.

```
for(i=0; i<NI; i++)  
  for(j=0; j<NJ; j++)  
    for(k=0; k<NK; k++)  
S:    A[i][j] += f(B[k][i], C[k][j]);
```

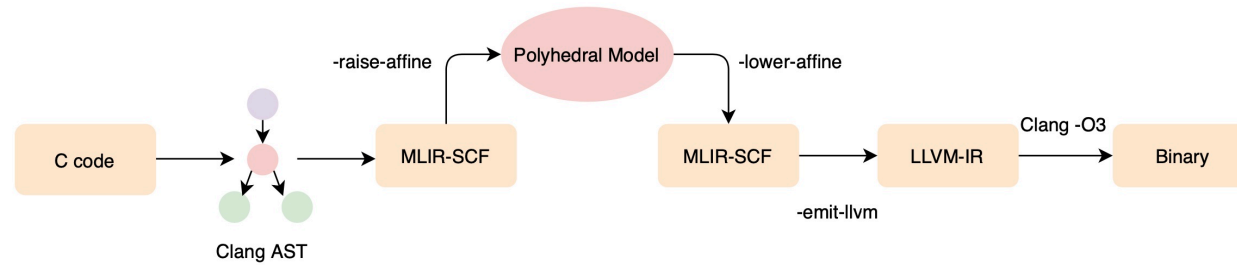
⇓

|                                                                                                                                                                                      |   |                                                                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <pre>for(i=0; i&lt;NI; i++)<br/>  for(j=0; j&lt;NJ; j++)<br/>    double M[NK];<br/>    for(k=0; k&lt;NK; k++)<br/>S:    M[k] = f(B[k][i], C[k][j]);<br/>T:    A[i][j] += M[k];</pre> | ⇒ | <pre>double M[NK];<br/>for(k=0; k&lt;NK; k++)<br/>  for(i=0; i&lt;NI; i++)<br/>    for(j=0; j&lt;NJ; j++)<br/>S:    M[k] = f(B[k][i], C[k][j]);<br/>T:    A[i][j] += M[k];</pre> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

# Statement Splitting



# Conclusion

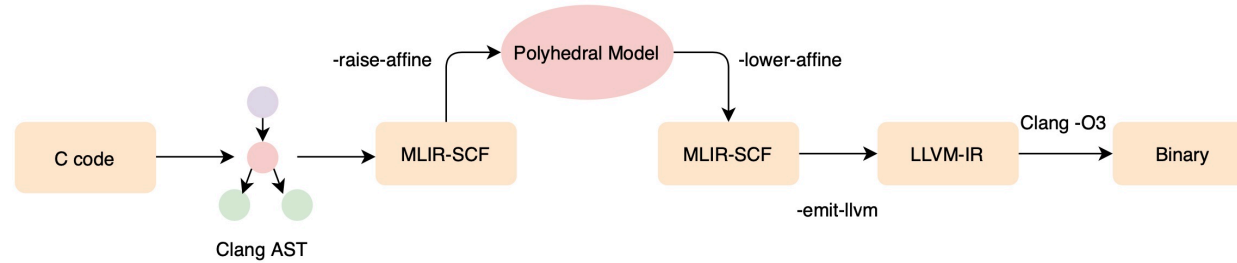


- Polygeist is an end-to-end polyhedral compiler that combines the best parts of source-level and compiler-level polyhedral tools
  - C/C++ frontend for MLIR
  - Transformations for raising to Affine
  - Integrating existing polyhedral tools with MLIR
- Polygeist provides an easy platform to introduce novel polyhedral optimizations (statement splitting, reduction detection) that are difficult to perform on existing representations
- Polygeist outperforms existing Polyhedral optimizers for both sequential and parallel code generation
- Open Sourced on Github, see <https://polygeist.mit.edu>

# Acknowledgements

- Thanks to Valentin Churavy, Albert Cohen, Henk Corporaal, Tobias Grosser, and Charles Leiserson for thoughtful discussions on this work.
- William S. Moses was supported in part by a DOE Computational Sciences Graduate Fellowship, in part by Los Alamos National Laboratories, and in part by the United States Air Force Research Laboratory.
- Lorenzo Chelini is partially supported by the European Commission Horizon 2020
- Ruizhe Zhao is sponsored by UKRI and Corerain Technologies Ltd. The support of the UK EPSRC is also gratefully acknowledged.

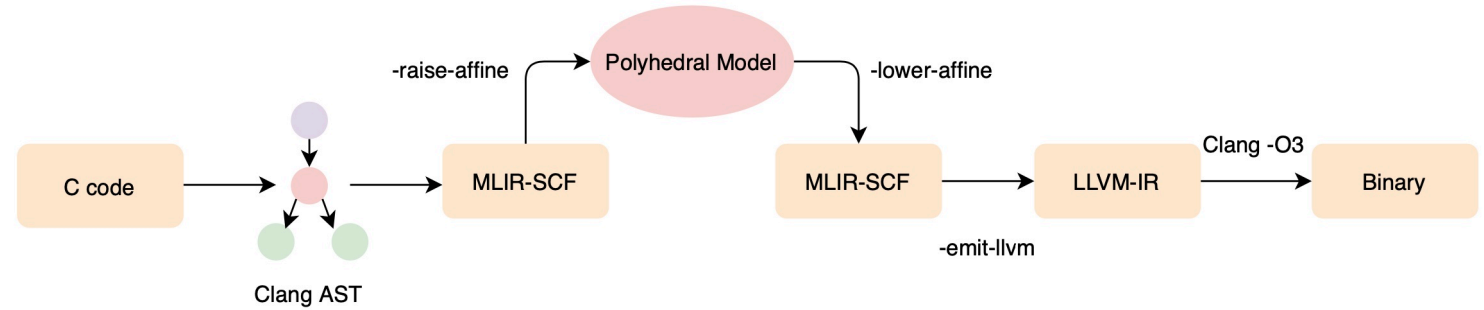
# Conclusion



- Polygeist is an end-to-end polyhedral compiler that combines the best parts of source-level and compiler-level polyhedral tools
  - C/C++ frontend for MLIR
  - Transformations for raising to Affine
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- Polygeist provides an easy platform to introduce novel polyhedral optimizations (statement splitting, reduction detection) that are difficult to perform on existing representations
- Polygeist outperforms existing Polyhedral optimizers for both sequential and parallel code generation
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Backup slides

# Future Work



- Exploration of Statement Splitting beyond a simple heuristic
- GPU optimization and GPU  $\leftrightarrow$  CPU
- Embedded DSL / C-style semantics for directly generating MLIR Ops
- Upstreaming LLVM Incubator Project

# Frontend Performance Differences

- 8% performance boost on Floyd-Warshall occurs if Polygeist generates a single MLIR module for both benchmarking and timing code by default
- MLIR doesn't properly generate LLVM datalayout, preventing vectorization for MLIR-generated code (patched in our lowering)
- Different choice of allocation function can make a 30% impact on some tests (adi)
- LLVM strength-reduction is fragile and sometimes misses reversed loop induction variable (remaining gap in adi)



# Backup Slides

```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  affine.for %arg2 = 0 to 10 {  
    affine.store %arg1, %arg0[%arg2 * 2] : memref<?xi32>  
  }  
  return  
}
```

# Conclusion

- Polygeist providing tools to fairly compare MLIR-based polyhedral representations with prior art in Polyhedral representations
  - C/C++ frontend for (Affine) MLIR
  - Integration of existing polyhedral tools for transforming MLIR (via OpenScop)
  - End-to-end comparison using existing Polyhedral benchmarks (Polybench)
- Polygeist enables future research on polyhedral MLIR transformations
- MLIR-based frontend differs from Clang by 1.25%
- @Ruizhe, add a good polymer conclusion

# Translate to OpenScop

- First pre-process MLIR Affine code by previous passes
- For each extracted polyhedral statement:
  - Domain: get constraints from affine.for/if
  - Initial Schedule: derive from region nesting and operation order
  - Access: extract from affine load/stores
- Store symbols in OpenScop extensions

# Translate to OpenScop

```
affine.for %i = 0 to %N
```

```
  affine.for %j = 0 to %N
```

```
    call @S0(%A, %i, %j)
```

```
func @S0(%A: memref<?x?xf32>, %i: index,
        %j: index) {
  %0 = affine.load %A[%i, %j]
  %1 = mulf %0, %0
  affine.store %1, %A[%i, %j]
  return
}
```

## Domain

| # | e/i | %i | %j | %N | 1  |                  |
|---|-----|----|----|----|----|------------------|
| 1 | 1   | 0  | 0  | 0  | 0  | ## %i >= 0       |
| 1 | -1  | 0  | 1  | -1 | -1 | ## -%i+%N-1 >= 0 |
| 1 | 0   | 1  | 0  | 0  | 0  | ## %j >= 0       |
| 1 | 0   | -1 | 1  | -1 | -1 | ## -%j+%N-1 >= 0 |

## Scattering

| # | e/i | s1 | s2 | s3 | s4 | s5 | %i | %j | %N | 1 |
|---|-----|----|----|----|----|----|----|----|----|---|
| 0 | -1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0 |
| 0 | 0   | -1 | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0 |
| 0 | 0   | 0  | -1 | 0  | 0  | 0  | 0  | 0  | 0  | 0 |
| 0 | 0   | 0  | 0  | -1 | 0  | 0  | 0  | 1  | 0  | 0 |
| 0 | 0   | 0  | 0  | 0  | -1 | 0  | 0  | 0  | 0  | 0 |

## READ/WRITE Accesses

| # | e/i | Arr | [1] | [2] | %i | %j | %N | 1 |       |
|---|-----|-----|-----|-----|----|----|----|---|-------|
| 0 | -1  | 0   | 0   | 0   | 0  | 0  | 0  | 0 | ## %A |
| 0 | 0   | -1  | 0   | 1   | 0  | 0  | 0  | 0 | ## %i |
| 0 | 0   | 0   | -1  | 0   | 1  | 0  | 0  | 0 | ## %j |

# Regenerate MLIR Code

- Obtain a CLooG AST from an optimized OpenScop representation
- Regenerate MLIR code by traversing AST
- OpenScop symbols will be translated to MLIR values or operations based on a maintained symbol table.

# Motivation

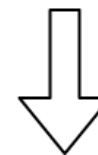
- ✓ The compiler research has recently been enamored by the MLIR framework, whose first-class polyhedral representation may provide benefits on a variety of codes
- ✓ We can fully leverage decades of polyhedral research by connecting MLIR with existing polyhedral tools.
- ✓ Without MLIR-versions of standard polyhedral benchmarks, one cannot perform a fair assessment
- ! Goal of this work is to provide a fair baseline for subsequent work AND explore the potential of polyhedral optimizations that require both high level and low level information

# Outlining

- Outline statements into functions
- Function interfaces:
  - Memory to access
  - Lifted stack allocated symbols

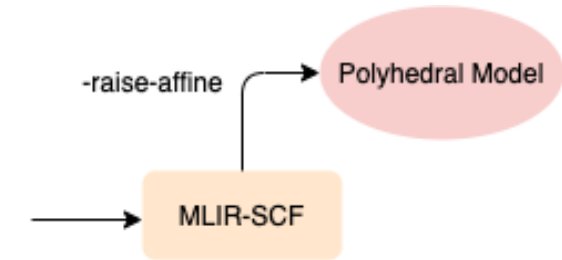
```
func @S0(%A: memref<?xf32>) {  
  %c0 = constant 0 : index  
  %s0 = dim %A, %c0 : index  
  
  %1 = affine.load %A[0]  
  affine.store %1, %A[symbol(%s0) - 1]  
  return  
}
```

Lift local symbols to the  
function interface



```
func @S0(%A: memref<?xf32>, %s0: index) {  
  %0 = affine.load %A[0]  
  affine.store %0, %A[%s0 - 1]  
  return  
}
```

# Polygeist Raising



- Select statements must be represented by a C ternary operator
  - C ternaries have lazy-evaluation semantics which are replicated in the generated MLIR
  - Mem2Reg and code motion attempt to remove unnecessary loads within if's to generate a valid select.

```
prefixMax[i] = (prefixMax[i-1] >= data[i])  
               ? prefixMax[i-1] : data[i];
```

```
%0 = index_cast %arg2 : i32 to index  
%1 = subi %0, %c1 : index  
%2 = load %arg0[%1] : memref<?xi32>  
%3 = load %arg1[%0] : memref<?xi32>  
%4 = cmpi "sgt", %2, %3 : i32  
%5 = scf.if %4 -> (i32) {  
  %6 = load %arg0[%1] : memref<?xi32>  
  scf.yield %6 : i32  
} else {  
  %6 = load %arg1[%0] : memref<?xi32>  
  scf.yield %6 : i32  
}  
store %5, %arg0[%0] : memref<?xi32>
```



# The Affine dialect

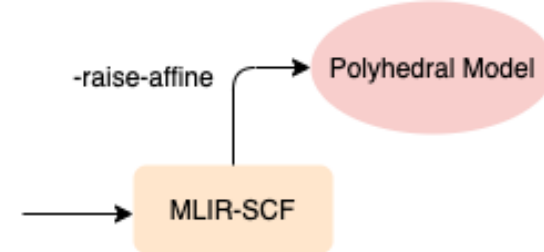
- Represent SCoP with polyhedral-friendly loops and conditions
- Core Affine representation
  - Symbols - parameters
  - Dimensions - symbol extension that accepts induction variables
  - Maps - multi-dimensional function of symbols and dimensions
  - Sets - integer tuples constrained by a conjunction

```
%c0 = constant 0 : index
%0 = dim %A, %c0 : memref<?xf32>
%1 = dim %B, %c0 : memref<?xf32>
affine.for %i = 0 to affine_map<() [s0] -> (s0)>()[%0] {
  affine.for %j = 0 to affine_map<() [s0] -> (s0)>()[%1] {
    %2 = affine.load %A[%i] : memref<?xf32>
    %3 = affine.load %B[%j] : memref<?xf32>
    %4 = mulf %2, %3 : f32
    %5 = affine.load %C[%i + %j] : memref<?xf32>
    %6 = addf %4, %5 : f32
    affine.store %6, %C[%i + %j] : memref<?xf32>
  }
}
```

# Polygeist Raising

```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  %c0 = constant 0 : index  
  %0 = alloca() : memref<1xmemref<?xi32>>  
  store %arg0, %0[%c0] : memref<1xmemref<?xi32>>  
  %1 = alloca() : memref<1xi32>  
  store %arg1, %1[%c0] : memref<1xi32>  
  %c0_i32 = constant 0 : i32  
  %c10_i32 = constant 10 : i32  
  %2 = index_cast %c10_i32 : i32 to index  
  scf.for %arg2 = %c0_i32 to %2 {  
    %3 = index_cast %arg2 : index to i32  
    %4 = alloca() : memref<1xi32>  
    store %3, %4[%c0] : memref<1xi32>  
    %5 = load %0[%c0] : memref<1xmemref<?xi32>>  
    %c2_i32 = constant 2 : i32  
    %6 = load %4[%c0] : memref<1xi32>  
    %7 = muli %c2_i32, %6 : i32  
    %8 = index_cast %7 : i32 to index  
    %9 = load %1[%c0] : memref<1xi32>  
    store %9, %5[%8] : memref<?xi32>  
  }  
  return  
}
```

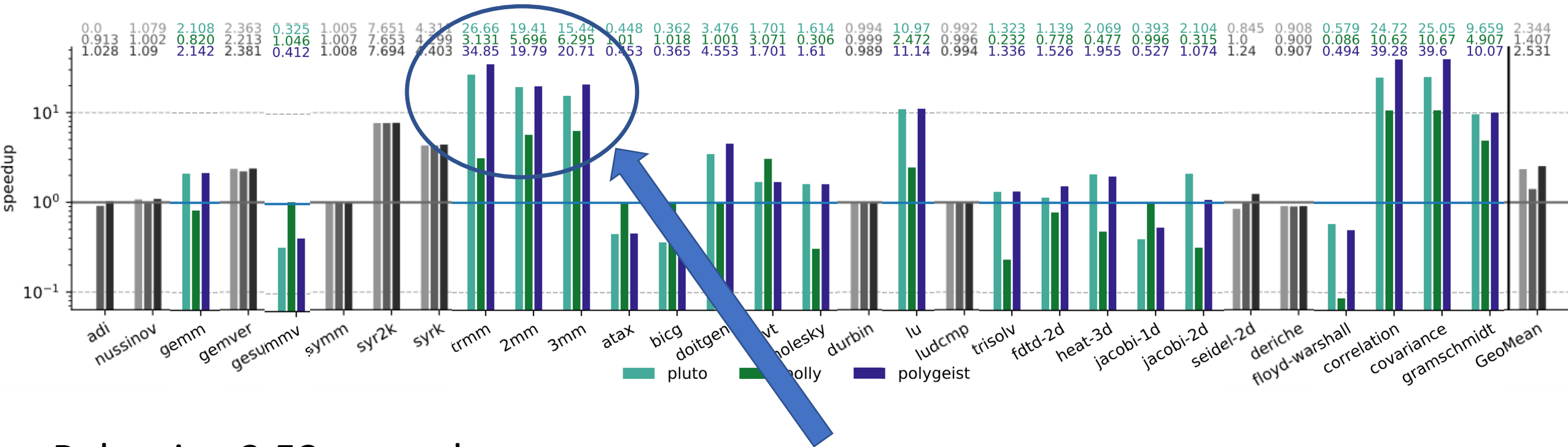
```
func @set(%arg0: memref<?xi32>, %arg1: i32) {  
  affine.for %arg2 = 0 to 10 {  
    affine.store %arg1, %arg0[%arg2 * 2]  
      : memref<?xi32>  
  }  
  return  
}
```



# Polyhedral Performance Differences

- Polly differs from other two as it uses a different scheduler
- Even when using the same scheduler, Polygeist can select a different statement set and thus schedule coming from partially optimized SSA rather than the original C.
- Pluto executes significantly more ( $\sim 10^{11}$ ) more integer instructions on seidel-2d than Polygeist, which is  $\sim 59$ s at 3GHz, accounting for the gap. Can be caused by different integer optimization and the use of a proper machine type/bound simplification.
- For jacobi-2d, Polygeist performs worse, stopping earlier when simplifying (75 statement copies in 40 branches), whereas Clang by default takes longer to process this but has better end vectorization.

# Sequential Polyhedral Comparison



Polygeist: 2.53x speedup  
Pluto: 2.34x speedup  
Polly: 1.41x speedup

Big gaps come from  
different schedules

# Parallel Performance Differences

- Same scheduling differences as sequential (Cholesky and LU are better on Pluto/Polygeist than Polly; Gemver and MVT are better on Polly)
- Ludcmp and syr(2)k benefit from SSA optimizations
- Polygeist is only framework that can parallelize deriche (6.9x) and symm (7.7x) by analyzing and removing the loop-carried dependency
- Polygeist identifies a parallel reduction within gramschmidt (56x Polygeist, 54x Pluto, 34x Polly) and durbin (6x slowdown as few iterations)